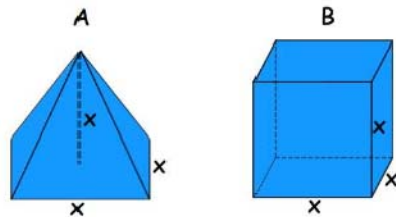Geometry CC – Mr. Valentino　　　　　　　　Name: _____

Unit 11 Lesson 9: Cavalieri's Principle and the Generatrix　　Date: _____　Per: _____

Aim: How can we relate 2D to 3D images?
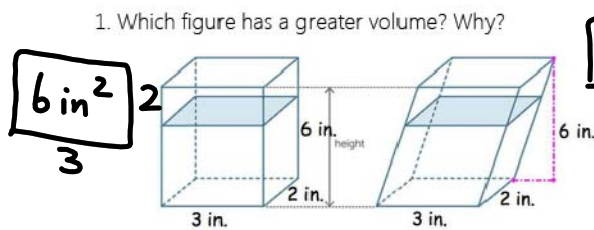
Do Now: Which figure has a larger volume? Why?

A　　　　　　　　　　　　B

B has a larger volume
(it's exactly 3x the volume of
_A)_

Cavalieri's Principle

If the areas of the cross sections of 3D shapes are equal and the shapes have the same height, then their volumes are equal.
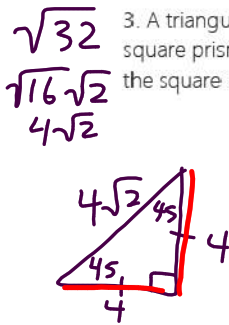
1. Which figure has a greater volume? Why?

$\boxed{6\,in^2}$ 2

3

6 in. height

2 in.

3 in.

$\boxed{6\,in^2}$ 2

3

6 in.

2 in.

3 in.

They have the same volume.
The area of their cross sections is the same.

2. Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder. How do their volumes compare?

Their volumes are the same.

$\sqrt{32}$
$\sqrt{16\sqrt{2}}$
$4\sqrt{2}$

3. A triangular prism has an isosceles right triangular base with a hypotenuse of $\sqrt{32}$ and a prism height of 15. A square prism has a height of 15 and its volume is equal to that of the triangular prism. What are the dimensions of the square base, in simplest radical form?

$4\sqrt{2}$

$A = \frac{1}{2}bh$
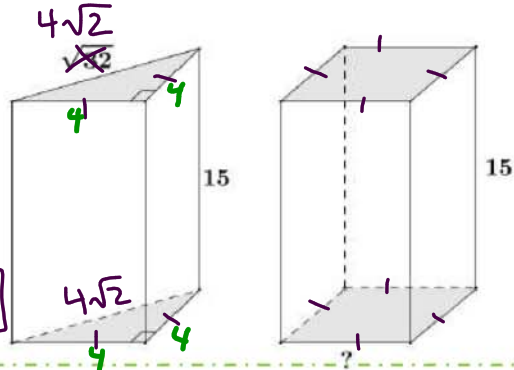
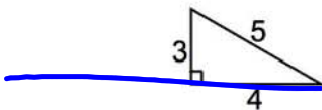$A = \frac{1}{2} \cdot 4 \cdot 4$ $\sqrt{s^2} = \sqrt{8}$

$A = 8$ $s = \sqrt{8}$

$= \sqrt{4\sqrt{2}}$

$= \boxed{2\sqrt{2}}$

$4\sqrt{2}$ $4\sqrt{45}$ 45 4

45
4

$4\sqrt{2}$
4
4
4

15

1
1
1
15
?

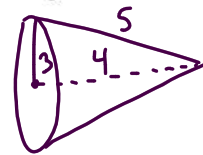### Generatrix

If you rotate the right triangle below about its altitude, what kind of 3-D figure could be formed? Draw it.

3D Figure:

CONE!

3 4 5
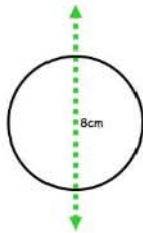
3 5 4

What are the dimensions of the new 3-D figure? Label them above.

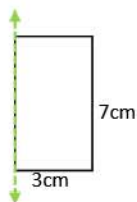What shape is created when revolving the figure around the given axis? ___Sphere___

What are the dimensions of the new shape?

$d = 8\, cm$

$r = 4\, cm$

8cm

What shape is created when revolving the figure around the given axis? ___cylinder___
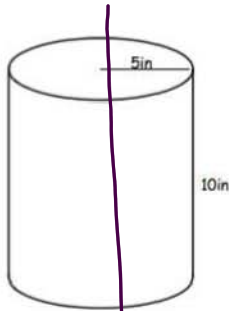
What are the dimensions of the new shape?

$h = 7\, cm$

$r = 3\, cm$

7cm

What if the location of the axis changed? What shapes could be created?

3cm

2

Practice Problems

1. If the cylinder is sliced **horizontally**, what are the dimensions of the cross section? Draw a picture of the cross section and label its dimensions.

$A =$ (circle with $.5$)

What is the area of the cross section in terms of pi?

$$A = \pi r^2$$
$$5^2 = 25\pi$$

(cylinder figure labeled 5in and 10in)

2. If the cylinder is sliced **vertically** through the middle, what are the dimensions of the cross section? Draw a picture of the cross section and label its dimensions.
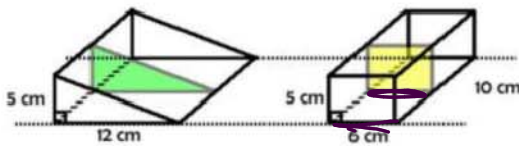
(rectangle labeled 10 and 10)

What is the area of the cross section?

3. What generatrix could be used to generate the cylinder above? Draw and label the dimensions of ALL the possibilities (make sure to include the axis of rotation)

(rectangle labeled 5 across top, 10 on left side, with axis of rotation on right)
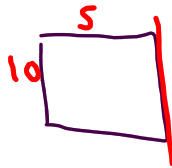
4. Jenny says that the two prisms DO NOT have the same volume because the cross sections are not the same shape. Renee disagrees; she says that it isn't the shape that has to be the same, it is the area that must be equal. Renee thinks they have the same volume. Who is right and why?
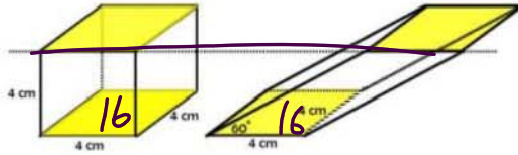
(two prism figures: left triangular prism labeled 5 cm, 12 cm; right rectangular prism labeled 5 cm, 10 cm, 6 cm)

$$\frac{1}{2}bh$$
$$\frac{1}{2}12 \cdot 5$$
$$6 \cdot 5$$
$$30$$

$$\ell \cdot w$$
$$6 \cdot 5$$
$$30$$

Who is correct? **Renee**

Explanation: **The area**
_____
_____
_____
_____

5. If the volume of the cube is 64 cm³, what is the volume of the oblique prism if it has been tilted at 60°?

4 cm

16

4 cm

16 cm.

4 cm

60°

4 cm

Volume:___**64 cm³**___
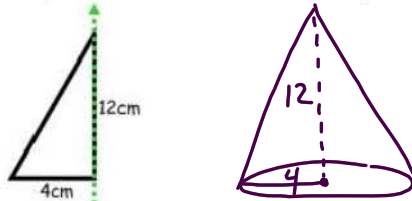
Explanation:_____

_____

_____

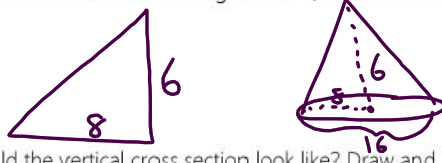6. Find the volume of the cone, to the nearest tenth.
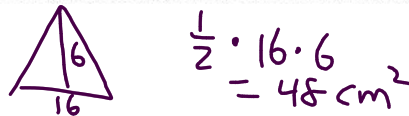
$$V = \tfrac{1}{3}\pi r^2 h$$

5.8 m

3.2 m

7. You are given the generatrix below. Draw the 3D shape created when rotating it about the axis. Label the dimensions.

12cm

4cm

12

4

8. If the base is doubled and the height halved, what are the dimensions of the new solid? Draw and label its dimensions:
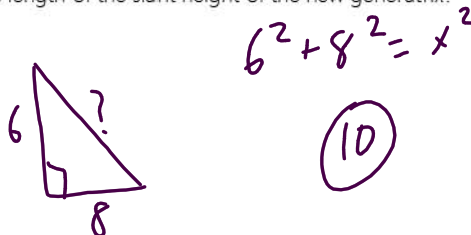
6

8

6

8

16

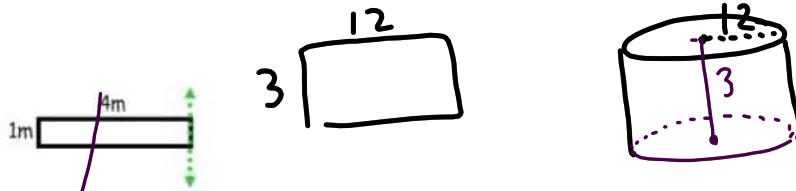9. What would the vertical cross section look like? Draw and label its dimensions. What's its area?

6

16

$$\tfrac{1}{2} \cdot 16 \cdot 6$$
$$= 48 \, cm^2$$

10. What could be the length of the slant height of the new generatrix?

6

?

8

$$6^2 + 8^2 = x^2$$

⑩

11. If the following generatrix was dilated by a scale factor of 3, what would be the result of rotating it around the given axis? Draw and label its new dimensions.

4m

1m

12

3

12

3

12. What would be the dimensions of the vertical and horizontal cross sections? Draw and label them below.

24

12

3

2D shapes
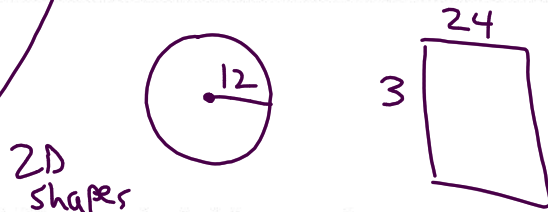
13. Find the areas of each of the cross sections.

$A = \pi (12)^2$

144π

$A = l \cdot w$