Dilation with Center NOT at Origin:

A dilation on a coordinate axis where the center is NOT the origin can be accomplished by observing the vertical and horizontal distances of each vertex from the center of dilation. In essence, we will be looking at the "slope" of each line (segment) involved.

Example:

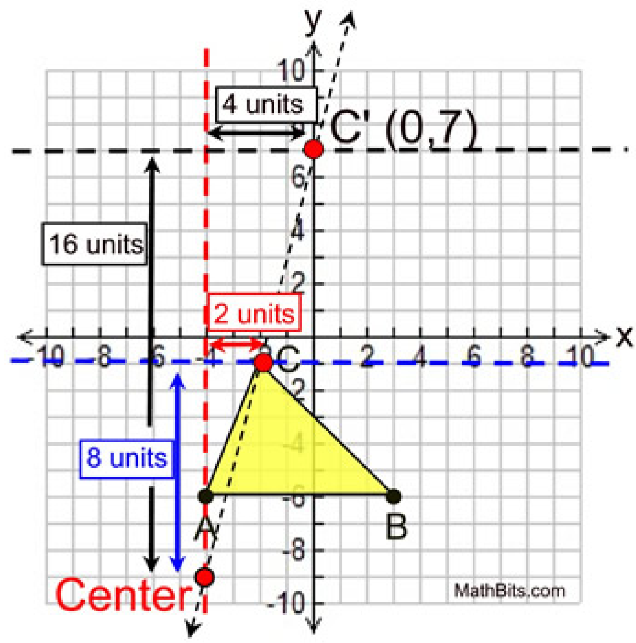
Part 1: Given point C (-2,-1), center of dilation of (-4,-9), and scale factor of 2, find C'.

By observation, point *C* is 8 vertical units above the center of dilation. Under a scale factor of 2, point *C′* needs to be 16 vertical units from the center.

Also, point *C* is 2 horizontal units right of the center of dilation. Point *C′* needs to be 4 horizontal units right of the center.

Starting at the center of dilation (-4,-9), move 16 units up and 4 units to the right to find *C′* at (0,7).

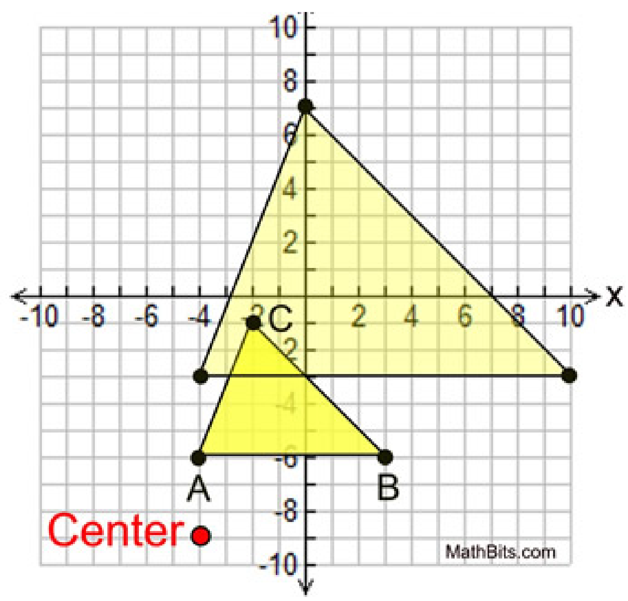
NOTE: We are maintaining the *slope* of the line passing through the center, point *C,* and point *C′*.



Part 2: Plot the dilation of ΔABC by a scale factor of 2 with a center of dilation at (-4,-9). A(-4,-6), B(3,-6), and C(-2,-1)

By observing vertical and horizontal distances from the center of dilation, as seen in Part 1, you can find the remaining two coordinates of the dilated triangle.

A'(-4,-3), B'(10,-3), C'(0,7)



The counting of vertical and horizontal distances shown above is a simple and easy way to find the coordinates for a dilation not centered at the origin.

FYI: Another Method

A dilation not centered at the origin, can also be thought of as a series of translations, and expressed as a formula. Translate the center of the dilation to the origin, apply the dilation factor as shown in the "center at origin" formula, then translate the center back (undo the translation).

- First translate the center of the dilation so the origin becomes the center.
  Subtracting the coordinate values of the center of dilation will move the center to the origin.
  Given center of dilation at (a,b), translate the center to $(0,0)$: $(x - a, y - b)$.

- Then apply the dilation factor, k: $(k(x - a), k(y - b))$

- And translate back: $(k(x - a) + a, k(y - b) + b)$

Formula: Center Not at Origin:

$$D_{O,k}(x,y) = (k(x-a)+a, k(y-b)+b)$$

O = center of dilation at (a,b); k = scale factor

Example:

Write a coordinate rule to find the vertices of a dilation with center (4,-2) and scale factor of 3.

Let (x,y) be a vertex of the figure.
Translate so the origin becomes center of the dilation (left 4 and up 2): $(x - 4, y + 2)$.
Apply the dilation formula when centered at origin: $(3(x - 4), 3(y + 3)) = (3x - 12, 3y + 9)$
Translate back (right 4 and down 2): $(3x - 12 + 4, 3y + 9 - 2) = (3x - 8, 3y + 7)$

Rule: $(x,y) \rightarrow (3x - 8, 3y + 7)$

For example, under this dilation, the point (5,6) becomes (3(5)-8, 3(6)+7) which is (7,25).