Name: KEY

Date:

Period:

Mr. Valentino

Unit 10 Review Sheet

Test Topics!

- Slope
- Midpoint
- Distance
- Quadrilateral Proofs
- Triangle Proofs

Slope

SLOPE FORMULA: $y = mx + b$

POINT SLOPE: $y - y_1 = m(x - x_1)$

- To decide if lines are parallel or perpendicular, first write the lines in slope intercept form, which is $y = mx + b$.
- Parallel lines have slopes that are the same.
- Perpendicular lines have slopes that are negative reciprocals.

State whether the lines are parallel, perpendicular, or neither.

$y = 6x - 3$

$y = -\frac{1}{6}x + 7$

⊥

2. $y = (3x) + 2$

$\frac{2y}{2} = \frac{6x - 6}{2}$

$y = (3x) - 3$

∥

3. $3x + 2y = 5$

$3y + 2x = -3$

$2y = -3x + 5$

$3y = -2x - 3$

$y = -\frac{3}{2}x + \frac{5}{2}$

$y = -\frac{2}{3}x - 1$

Neither

4. What is the slope of the line passing through the points (-5, 6) and (4, -3)?

$\frac{\Delta y}{\Delta x} = \frac{-3 - 6}{4 + 5} = \frac{-9}{9} = -1$

5. What is the slope of the line parallel to the line in question 4? -1

6. What is the slope of the line perpendicular to the line in question 4? 1

7. A line u passes through (6, 1) and (8, p). A line v passes through (2, −3) and (10, −6). The lines u and v are parallel. Find the value of p.

$\frac{\Delta y}{\Delta x} = \frac{-6 + 3}{10 - 2} = \frac{-3}{8}$

$p = \frac{1}{4}$

$y - 1 = -\frac{3}{8}(x - 6)$

$y - 1 = -\frac{3}{8}x + \frac{18}{8}$

$y = -\frac{3}{8}x + \frac{26}{8}$ ← eq. of line

$y = -\frac{3}{8}x + \frac{13}{4}$

$y = -\frac{3}{8}(8) + \frac{13}{4}$

$y = -\frac{24}{8} + \frac{26}{8}$

$y = \frac{2}{8} = \boxed{\frac{1}{4}}$

8. What is the equation of the line passing through the points A(4, -5) and B(-2, -2)?

$\frac{\Delta y}{\Delta x} = \frac{-2 + 5}{-2 - 4} = \frac{3}{-6} = -\frac{1}{2}$

$y + 5 = -\frac{1}{2}(x - 4)$

$y + 5 = -\frac{1}{2}x + 2$

$\boxed{y = -\frac{1}{2}x - 3}$

Midpoint

MIDPOINT FORMULA: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

1. What is the midpoint of line segment AB with A(11, -5) and B(1, -10)?

$\left(\frac{11+1}{2}, \frac{-5-10}{2}\right)$

$\left(\frac{12}{2}, \frac{-15}{2}\right) \rightarrow \left(6, \frac{-15}{2}\right)$

2. What is the midpoint of line segment AB with A(14, 18) and B(-6, 10)?

$\left(\frac{14-6}{2}, \frac{18+10}{2}\right)$

$\left(\frac{8}{2}, \frac{28}{2}\right) \rightarrow (4, 14)$

3. What is the endpoint of line segment AB given A(4, 6) and midpoint M(-3, -2)?

x: $4 \xrightarrow{(-7)} -3 \xrightarrow{(-7)} -10$

y: $6 \xrightarrow{(-8)} -2 \xrightarrow{(-8)} -10$

$(-10, -10)$

4. What is the equation of the perpendicular bisector of line segment AB with endpoints A(-4, -2) and B(8, 4)?

① Find midpoint

② Find slope ↓ then identify neg. reciprocal

$\left(\frac{-4+8}{2}, \frac{-2+4}{2}\right)$

$\left(\frac{4}{2}, \frac{2}{2}\right)$

$(2, 1)$

$\frac{\Delta y}{\Delta x} = \frac{4+2}{8+4} = \frac{6}{12} = \frac{1}{2}$ ↓ neg. reciprocal -2

$y = mx + b$

$1 = -2(2) + b$

$1 = -4 + b$

$b = 5$

$y = -2x + 5$

5. What is the equation of the perpendicular bisector of line segment AB with endpoints A(-9, 11) and B(-15, 19)?

$\left(\frac{-9-15}{2}, \frac{11+19}{2}\right)$

$\left(\frac{-24}{2}, \frac{30}{2}\right)$

$(-12, 15)$

$\frac{\Delta y}{\Delta x} = \frac{19-11}{-15+9} = \frac{8}{-6} = -\frac{4}{3}$ ↓ neg. reciprocal $\frac{3}{4}$

$y = mx + b$

$15 = \left(\frac{3}{4}\right)(-12) + b$

$15 = -9 + b$

$b = 24$

$y = \frac{3}{4}x + 24$

Distance

DISTANCE FORMULA: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

1. Where does the distance formula originate from? pythagorean theorem

2. What is the length of the line segment connecting A(5, 9) and B(-7, -7)?

$\sqrt{(-7-5)^2+(-7-9)^2}$

$\sqrt{(-12)^2+(-16)^2}$

$\sqrt{144+256} = \sqrt{400}$

$= 20$

3. What is the length of the line segment connecting A(3, 8) and B(9, 10)?

$\sqrt{(9-3)^2+(10-8)^2}$

$\sqrt{(6)^2+(2)^2}$

$\sqrt{36+4} = \sqrt{40}$

. The point (-3,-6) lies on a circle. What is the length of the radius of this circle if the center is located at (9,-2)?

$\sqrt{(9+3)^2+(-2+6)^2}$

$\sqrt{(12)^2+(4)^2}$

$\sqrt{144+16}$

$\sqrt{160} = 4\sqrt{10}$

5. The point (10,-5) lies on a circle. What is the length of the diameter of this circle if the center is located at (6,4)?

$\sqrt{(6-10)^2+(4+5)^2}$

$\sqrt{(-4)^2+(9)^2}$

$\sqrt{16+81}$

$\sqrt{97} \longrightarrow 2\sqrt{97}$

Quadrilateral Proofs

1. Prove that the quadrilateral with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.

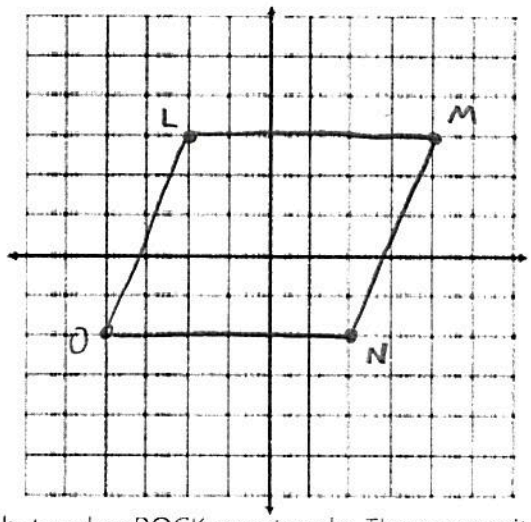
slope LM = 0 LM ∥ ON b/c they have the same slope.

slope ON = 0

slope LO = $\frac{-2-3}{-4+2} = \frac{-5}{-2} = \frac{5}{2}$ LO ∥ MN b/c they have the same slope.

slope MN = $\frac{-2-3}{2-4} = \frac{-5}{-2} = \frac{5}{2}$

LMNO is a P b/c it has 2 pairs of ∥ sides.



2. Given the points R(-1, 0), C(-3, -4) and K(0, -1), find the point O that makes ~~ROCK~~ RCOK a rectangle. Then prove it is a rectangle.

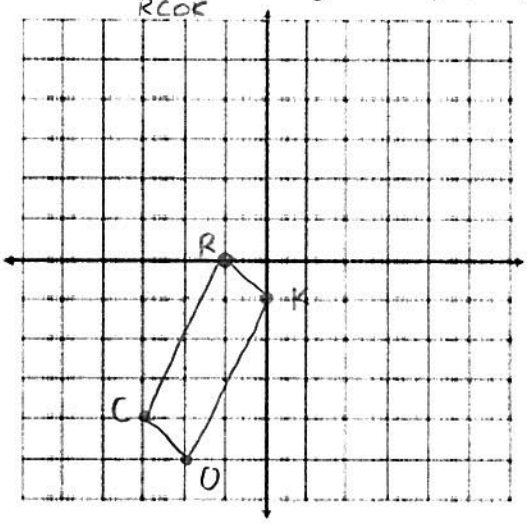
To find 4th point:

$\left(\frac{-3+0}{2}, \frac{-4-1}{2}\right)$

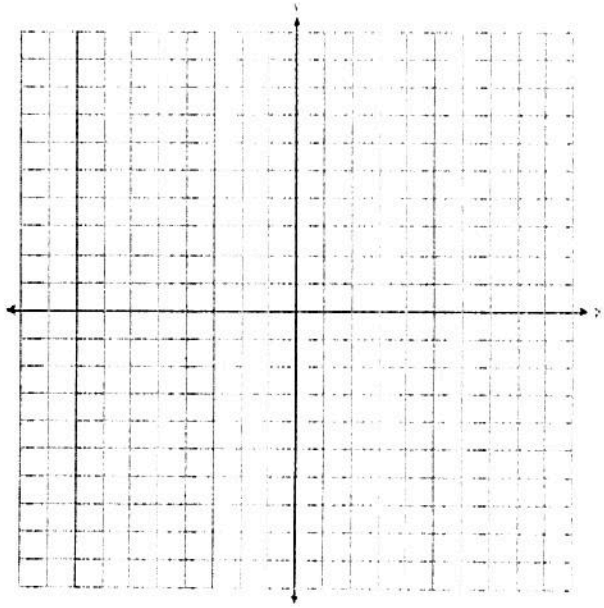
$\left(\frac{-3}{2}, \frac{-5}{2}\right)$

(-1.5, -2.5)

(-2, -5)



~~3.~~ Given Rhombus GHJK with G(-5, 5), H(0, 3) and K(-7, 0). Find the coordinates of J. Then prove GHJK is a rhombus.



4. Prove that the quadrilateral with vertices A(-1,0), B(3,3), C(6,-1) and D(2,-4) is a square.

slope AB: $\frac{3-0}{3+1} = \frac{3}{4}$

slope DC: $\frac{-4+1}{2-6} = \frac{-3}{-4} = \frac{3}{4}$

AB ∥ DC b/c they have the same slope.

slope AD: $\frac{-4-0}{2+1} = \frac{-4}{3}$

slope BC: $\frac{-1-3}{6-3} = \frac{-4}{3}$

AD ∥ BC b/c they have the same slope.

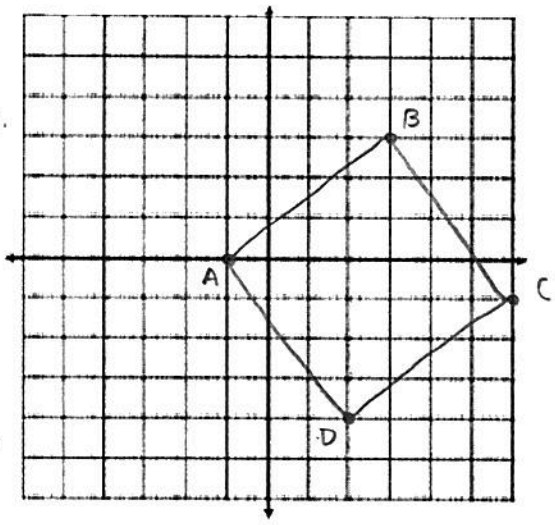
ABCD is a ▱ b/c opp sides are ∥.

ABCD is a R b/c AB ⊥ AD which means there is a right ∡ b/c ⊥ lines form right ∡'s.

$AB = \sqrt{(3+1)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(3)^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

ABCD is a square b/c it is a R w/ ≅ adj. sides.



5. Prove that quadrilateral MILK with the vertices M(1,3), I(-1,1), L(-1, -2), and K(4,3) is an isosceles trapezoid.

slope IM: $\frac{3-1}{1+1} = \frac{2}{2} = 1$

slope LK: $\frac{3+2}{4+1} = \frac{5}{5} = 1$

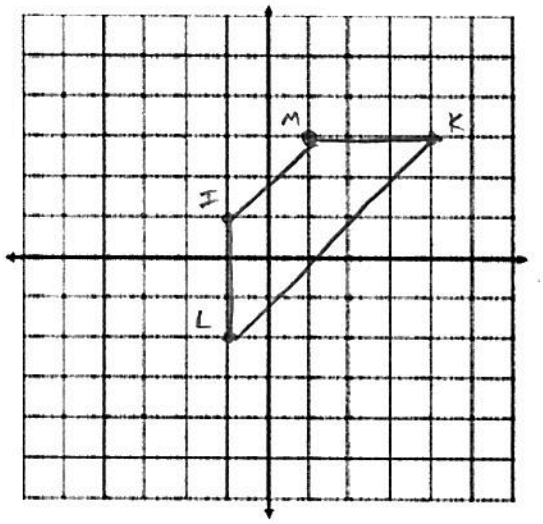
MILK is a trapezoid b/c it has at least 1 pair of ∥ sides.

IL = 3

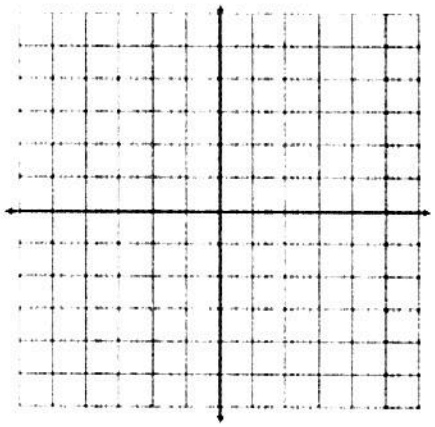
MK = 3

IL ≅ MK b/c they have the same length

MILK is a iso. trapezoid b/c the legs are ≅.



6. In parallelogram MATH, the coordinates of the endpoints of the diagonal MT are M (1, 3) and T (5, 1). Which of the following equations contains diagonal AH and would prove MATH is a rhombus?



~~1)~~ $y = -\frac{1}{2}x + 2$

2) $y = 2x - 4$

~~3)~~ $y = -\frac{1}{2}x + 6$

4) $y = 2x + 3.5$

$\frac{\Delta y}{\Delta x} = \frac{3-1}{1-5} = \frac{2}{-4} = -\frac{1}{2} \longrightarrow 2$

$\left(\frac{1+5}{2}, \frac{3+1}{2}\right)$

$\left(\frac{6}{2}, \frac{4}{2}\right) \rightarrow (3,2)$

$y = mx + b$

$2 = 2(3) + b$

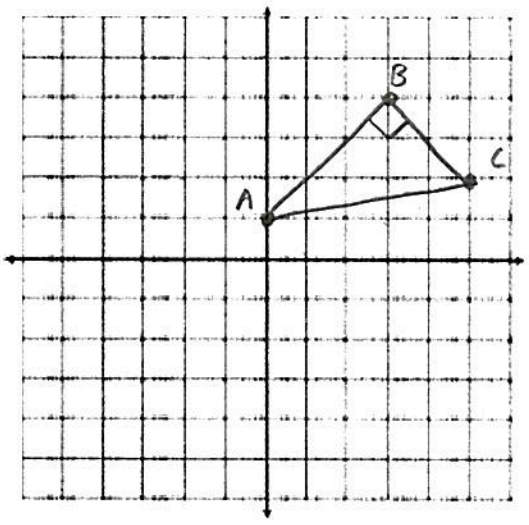
$2 = 6 + b$

$b = -4$

$y = 2x - 4$

Triangle Proofs

7. Prove that $A(0, 1)$, $B(3, 4)$, $C(5, 2)$ is a right triangle.



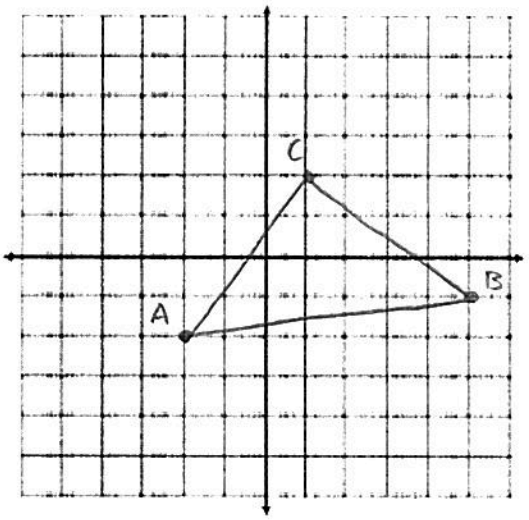
slope $AB = \frac{4-1}{3-0} = \frac{3}{3} = 1$

slope $BC = \frac{4-2}{3-5} = \frac{2}{-2} = -1$

$AB \perp BC$ b/c they have neg. reciprocal slopes.

$\triangle ABC$ is a right $\triangle$ b/c it has a right $\measuredangle$.

8. Prove that $A(-2, -2)$, $B(5, -1)$, $C(1, 2)$ is an isosceles triangle.



$AC = \sqrt{(-2-1)^2 + (-2-2)^2}$

$= \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(5-1)^2 + (-1-2)^2}$

$= \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$AC \cong BC$ b/c they have the same length.

$\triangle ABC$ is isosceles b/c it has 2 $\cong$ sides.