Name: _____   Date: _____

SYSTEMS UNIT, LESSON 1: SOLUTIONS TO LINEAR SYSTEMS AND SOLVING BY GRAPHING

Systems of equations (and inequalities) are essential to modeling situations with **multiple variables and multiple relationships between the variables**. At the end of the day, though, the solution set of a system of equations can be easily defined:

SOLUTIONS TO A SYSTEM OF EQUATION

1. A point (x, y) is a **solution** to a system if it makes **all equations true**.

2. The **solution set** of a system is the collection of **all** pairs (x, y) that are solutions to the system (see 1).

DO NOW

Exercise #1: Determine if the point $(2, 5)$ is a solution to each of the systems provided. Show the work that leads to your answer for each.

x   y

(a) $y = 4x - 3$ $\longrightarrow$ $5 = 4(2) - 3$
$$5 = 8 - 3$$
$$5 = 5$$

$2x + y = 9$

$2(2) + 5 = 9$
$$4 + 5 = 9$$
$$9 = 9$$ YES!

(b) $y - x = 3$ $\longrightarrow$ $5 - 2 = 3$
$$3 = 3$$

$y = \frac{1}{2}x + 6$

$5 = \frac{1}{2}(2) + 6$
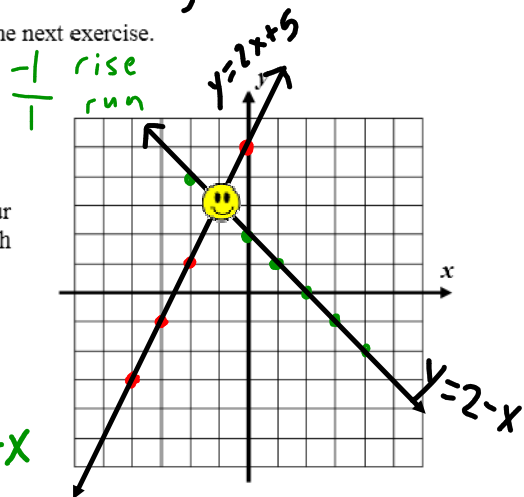$$5 = 1 + 6$$
$$5 = 7 \quad X$$

$(2, 5)$ is NOT a solution.

We can solve a system by using a graph. Review this process in the next exercise.

Exercise #2: Consider the system of equations shown below:

① $y = 2x + 5$

② $y = 2 - x$ $\longrightarrow$ $y = -x + 2$

$-\frac{1}{1}$ rise / run

(a) Graph both equations on the grid shown. Use **TABLES** on your calculator to make the process faster, if necessary. Label each line with its equation.

(b) At what point do the two lines intersect?

x   y

$(-1, 3)$

(c) Show that this point is a solution to the system.

① $y = 2x + 5$
$$3 = 2(-1) + 5$$
$$3 = -2 + 5$$
$$3 = 3 \checkmark$$

② $y = 2 - x$
$$3 = 2 - (-1)$$
$$3 = 2 + 1$$
$$3 = 3 \checkmark$$

y = 2x + 5

y = 2 - x

1

It's easy to see why the method of graphing works if you understand the **truth about graphs**. Remember:

> **GRAPHS OF EQUATIONS**
>
> 1. A point (x, y) lies on a graph of an equation if it makes that equation **true**.
>
> 2. The graph of an equation is simply the set of all points (x, y) that make the equation **true**.

Exercise #3: So, now you can put the definition of the graph of an equation together with the definition of a system. Fill in the blanks with one of the words shown:

TRUE, INTERSECTION, SOLUTIONS, BOTH

(handwritten in red: Not true, 3 ≠ 2)

1. To solve a **system of equations graphically** you find the __intersection point__ of the two graphs.

2. This works because any intersection point must lie on __both__ graphs.

3. Because intersection points lie on both graphs, they must make both equations __Correct → (true)   3=3 ✓__.

4. Because intersection points make both equations true, they are __Solutions__ to the system of equations.

We will often use this graphical method to solve systems in applied problems. Let's take a look at a modeling problem involving a linear system of equations.

Exercise #4: Janelle and Swetha are taking a 50 question true false test in their history class. Janelle started after Swetha had already finished 12 questions. Janelle answers questions at a rate of two per minute, while Swetha answers them at a rate of __5 questions every 4 minutes.__ Janelle eventually catches up to Swetha. How many minutes does it take her and what question are they on when Janelle catches up?

(handwritten: 10 q → 8 min)

(a) Create two linear models for Janelle and Swetha's questions answered since Janelle started. It may help to plot some points on the graph paper. Show the work that you use.

Janelle: $y = 2x$

Swetha: $y = \frac{5}{4}x + 12$
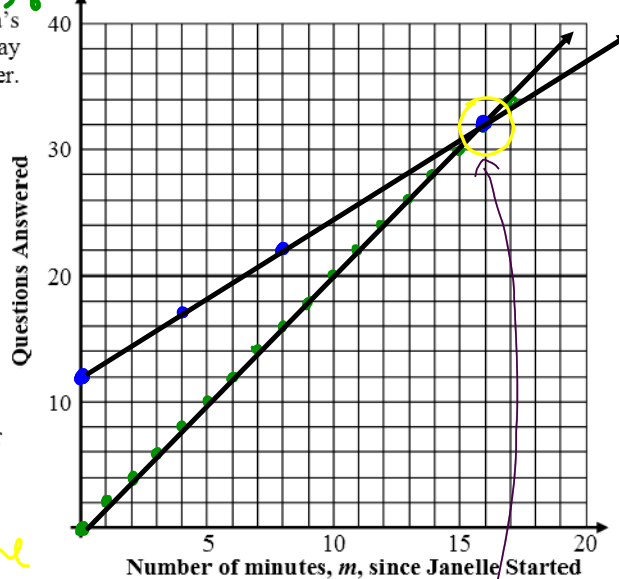
(b) Graph the two equations, using your calculator as needed, and solve the problem.

★ *16 minutes she catches up at question #32*

(handwritten near graph: (16, 32), min, questions answered)

Questions Answered vs. Number of minutes, *m*, since Janelle Started

Name: _____   Date: _____

SOLUTIONS TO SYSTEMS AND SOLVING BY GRAPHING
HOMEWORK

1. Determine whether each of the following points is a solution to the given system. Justify your answer.

(a) $(3, 4)$

$x + y = 7$

$y = 2x - 2$

(b) $(-10, -1)$

$y = \dfrac{1}{2}x + 4$

$y = 4x + 30$

(c) $(2, 14)$

$y = -3x + 20$

$y = 2x + 10$

(d) $\left(2, \dfrac{3}{2}\right)$
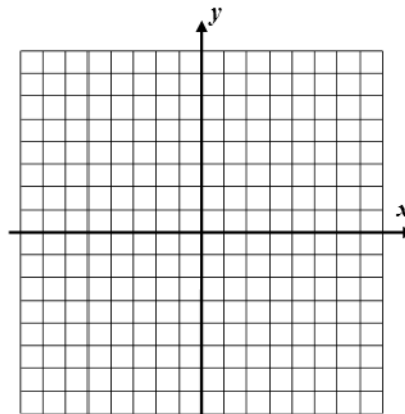
$y = \dfrac{8 - x}{4}$

$y = \dfrac{5}{4}x - 1$

OVER

3

2. Solve the following system of equations graphically. After graphing, be sure to label each line with its equation and state your final solution as a coordinate pair.

$y = \dfrac{1}{3}x + 1$

$x + y = 5$

3. Which of the following points solves the system shown below?

(1) $(1, -4)$ (3) $(2, 8)$ $y = 5x - 9$

(2) $(3, 6)$ (4) $(-3, 18)$ $y = -2x + 12$