Name: ______________________________ Date: ________________

Exponential Unit, Lesson 2: Zero and Negative Exponents

In math, people often invent ways to **extend concepts** to areas that might not make sense at first. Pretty much everyone can understand what 2^3 means, because they understand that it represents multiplying by the number 2, 3 times. Yet, what does 2^0 or 2^{-4} mean? Does it make sense to talk about multiplying by a number a negative amount of times? Let's explore these ideas in the first exercise.

***Do Now (a)*:** We can think of powers of 2 as representing multiplication of the number 1 repeatedly.

(a) Fill in the pattern for powers that are not negative. What does this lead you to fill in for 2^0?

$2^4 =$

$2^3 =$

$2^2 = 1 \cdot 2 \cdot 2 = 4$

$2^1 = 1 \cdot 2 = 2$

$2^0 =$

(b) **If positive exponents** indicated **multiplying** the number **1** by 2 repeatedly, then **negative exponents** should indicate ______________.

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2 \cdot 2} = \frac{1}{2^2} = \frac{1}{4}$

$2^{-3} =$

$2^{-4} =$

We want the pattern of positive, integer powers to extend to zero exponents and negative, integer exponents. We can now define zero and negative exponents as follows.

Zero and Negative Exponents

1. Zero Exponents:

2. Negative Exponents:

***Exercise* #2:** Which of the following is **not** equivalent to 5^{-2}?

(1) $\frac{1}{5^2}$ (3) $\frac{1}{25}$

(2) $\frac{1}{10}$ (4) 0.04

***Exercise* #3:** If $f(x) = 3x^{-2} + 2x^0$, then which of the following is the value of $f(2)$? Show the work that leads to your answer. Remember, exponents **always** come before multiplication.

(1) $2\frac{3}{4}$ (3) $1\frac{1}{12}$

(2) $1\frac{3}{4}$ (4) $2\frac{1}{2}$

Because we now have negative exponents we can develop a third **exponent law**. Recall that we already have the following two.

"Power to a Power"

EXPONENT LAWS (SO FAR)

1. $x^a \cdot x^b = x^{a+b}$ 2. $(x^a)^b = x^{a \cdot b}$

Now, let's see if we can develop a rule for dividing quantities that have the same base.

***Exercise* #4:** Rewrite each of the following expressions in simplest exponential form.

Recap (a) $\frac{x^5}{x^2} = x^3$ (b) $\frac{3^{10}}{3^5} = 3^5$ (c) $\frac{x^8}{x^2} = x^6$

(d) So it appears that: $\frac{x^a}{x^b} = x^{a-b}$

Now we have a **pattern** that works quite well if the **exponent** in the **numerator** is **greater** than that of the **denominator**. But does it work if that isn't true?

***Exercise* #5:** Rewrite each of the following expressions.

(a) $\frac{2^4}{2^4}$ $2^{4-4} = 2^0 = 1$

(b) $\frac{x^2}{x^7}$ $x^{2-7} = x^{-5} = \boxed{\frac{1}{x^5}}$

(c) $\frac{5^6}{5^{10}}$ 5^{6-10} $5^{-4} = \frac{1}{5^4} = \boxed{\frac{1}{625}}$

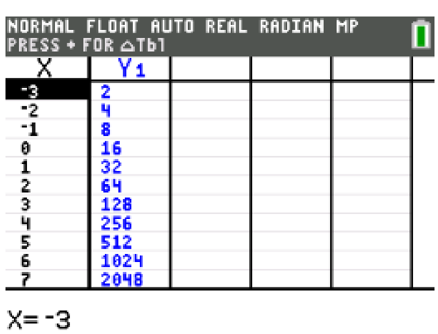
P
E
M
D
A
S

So, we now we see that the **subtraction rule for exponents** is consistent with negative and zero exponents. For now, we just want to be comfortable that negative exponents indicate division and positive exponents indicate multiplication.

***Exercise* #6:** Consider the **exponential function** $f(x) = 16(2)^x$. Find each of the following without your calculator.

(a) $f(0) = 16$ (b) $f(2) = 64$ (c) $f(-2) = 4$

$16 \cdot (2)^0$
$16 \cdot 1$
16



$16 \cdot (2)^{-2}$

$\frac{16}{1} \cdot \frac{1}{4} = \frac{16}{4}$

$= 4$

$2x + 2x = 4x$

$2y + 2x$

$x^5 + x^2$

$x^5 \cdot x^2 = x^7$

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ZERO AND NEGATIVE EXPONENTS
HOMEWORK

1. Rewrite each of the following as equivalent expressions without the use of negative or zero exponents. Remember your order of operations.

(a) 5^{-3} (b) 6^0 (c) 2^{-5}

(d) $4x^0$ (e) $(4x)^0$ (f) $x^{-2}y^4$

2. Which of the following is not equivalent to 2^{-3}?

(1) $\frac{1}{2^3}$ (3) 0.125

(2) -6 (4) $\frac{1}{8}$

3. If $f(x)=12(2)^x$, then which of the following represents the value of $f(-2)$?

(1) -48 (3) 3

(2) 6 (4) -4

4. If the expression $8(x+11)^0-2x^0+6x$ is evaluated when $x=-1$, the result would be

(1) 1 (3) 7

(2) 0 (4) 4

5. The numerical expression $\dfrac{\left(5^3\right)^2}{\left(5^2\right)^4}$ is equivalent to

(1) $\frac{1}{25}$ (3) 10

(2) 25 (4) $-\frac{1}{10}$

6. Write each of the following in the form ax^n, where n can be either a positive or negative integer.

(a) $\frac{x^3}{x^8}$ (b) $\frac{6x}{2x^8}$ (c) $\frac{28x^6}{21x^2}$

Applications

7. The number of people, n, who know a rumor can be modeled using the equation $n(d) = 20(2)^d$, where d is the number of days *since* Monday.

(a) Explain why $n(0) = 20$. What does this represent in terms of the situation modeled?

(b) What is the value of $n(-2)$? What does this represent in terms of the situation modeled?