Name: ______________________________ Date: ______________

EXPONENTIAL GROWTH AND DECAY

There are many things in the real world that grow faster as they grow larger or decrease slower as they get smaller. These types of phenomena, loosely speaking, are known as **exponential growth (and decay in the case of decreasing).** In today's lesson, we will look at both growth and decay.

***Exercise* #1:** The number of people who have heard a rumor often grows exponentially. Consider a rumor that starts with 3 people and where the number of people who have heard it doubles each day that it spreads.

(a) Why does it make sense that the number of people who have heard a rumor would grow exponentially?

The more days that go by, the more people there will be that hear the rumor.

(b) Fill in the table below for the number of people, N, who knew the rumor after it has spread a certain number of days, d.

d	0	1	2	3	4	5
N	3	6	12	24	48	96

(c) Could we figure out how many people would know the rumor after 20 days? How many?

3,145,728

(d) Is there a formula we could use to find the number of people that heard the rumor for any amount of days?

$f(x) = a(b)^x$

$a \rightarrow$ initial amount

$b \rightarrow$ growth factor

$x \rightarrow$ # of trials (times)

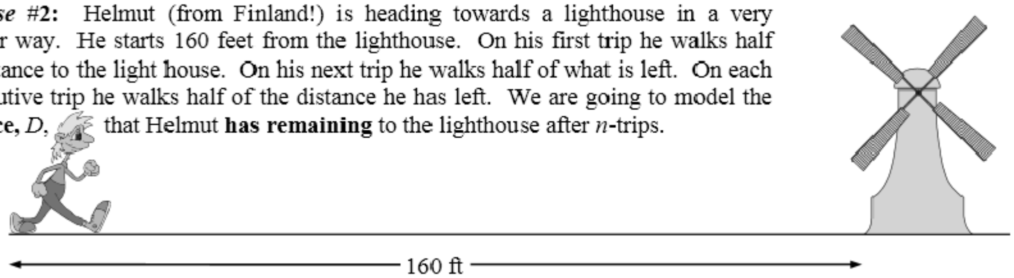
$3(2)^{20}$

$\downarrow$

3,145,728

Let's now look at developing a fairly simple exponential decay problem.

***Exercise* #2:** Helmut (from Finland!) is heading towards a lighthouse in a very peculiar way. He starts 160 feet from the lighthouse. On his first trip he walks half the distance to the light house. On his next trip he walks half of what is left. On each consecutive trip he walks half of the distance he has left. We are going to model the **distance, *D*,** that Helmut **has remaining** to the lighthouse after n-trips.



(a) Fill in the table below for the amount of distance that Helmut has left after n-trips.

n	0	1	2	3	4
D (ft)	160	80	40	20	10

(b) Each entry in the table could be found by **multiplying** the previous by what number? This is important because we **always** want to think about exponential functions in terms of **multiplying.**

Multiplying by $\frac{1}{2}$

(c) Like in Exercise #2(a), we want to see this process as repeated multiplication by $\frac{1}{2}$. Fill out each of the following pattern:

$n=0 \quad D=160$

$n=1 \quad D=160\cdot\frac{1}{2}=80$

$n=2 \quad D=80\cdot\frac{1}{2}=\left(160\cdot\frac{1}{2}\right)\cdot\frac{1}{2}=$

$n=3 \quad D=$

$n=4 \quad D=$

(d) Based on (c), give a formula that predicts the distance, D, that Helmut has left after n-trips.

$f(x)=a(b)^x \qquad 160\left(\frac{1}{2}\right)^x$

(e) How far is Helmut from the windmill after 6 trips? Provide a calculation that justifies your answer and don't forget those units!

(f) Helmut believes he will reach the windmill after 10 trips. Is he correct?

(g) Explain why Helmut will never reach the windmill?

Since Helmut will only walk half the distance left on each trip, he will always have some distance left.

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Exponential Growth and Decay
Homework

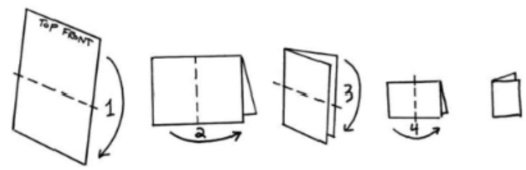
Applications

1. A piece of paper is 0.01 centimeters (cm) thick. When you fold it once, it becomes 0.02 centimeters thick. If you fold it again, it doubles again to 0.04 centimeters thick. Each fold doubles the thickness of the paper.

(a) How thick is the paper after:

4 Folds:

5 Folds:



(b) For each of the following number of folds, *f*, show how you can calculate the thickness, *T*, based on repeatedly multiplying by 2.

$f=0 \quad T=0.01$

$f=1 \quad T=0.01(2)^1=0.02$

$f=2 \quad T=0.02(2)=0.01(2)(2)=0.01(2)^2$

$f=3 \quad T=$

$f=4 \quad T=$

(c) Determine a formula, based on (b), for the thickness, *T*, based on the number of folds, *f*.

(d) How thick would the paper be if $f=10$? Use proper units

(e) If there are 100 centimeters in a meter, how many *meters* thick is the paper after 20 folds? Show the work that leads to your answer.

(d) If there are 1000 meters in a kilometer and the Moon is 384,000 kilometers away from the Earth, will the paper reach the Moon after 40 folds? Show the calculations that lead to your answer.