Name: _____   Date: _____

Sequences Day 2: Modeling Sequences!

Do Now:

During halftime at a football game, a high school marching band marches onto the field to perform a routine. There is one performer in the first row, four performers in the second row, and seven performers in the third row. This pattern continues for n rows.

(a) Create a sequence that represents the number of performers in each row.

$$\underline{1}, \underline{4}, \underline{7}, \underline{10}, \ldots$$

(b) Write an equation that can be used to find the number of performers in the nth row.

$$a_n = 1 + 3(n-1)$$

(c) How many performers are in the 6th row?

$$a_6 = 1 + 3(6-1)$$
$$a_6 = 1 + 3(5) = \boxed{16}$$

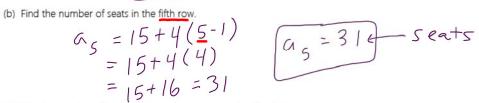## Problem Solving and Arithmetic Explicit Formulas

1) The first row of a theater has 15 seats in it. Each subsequent row has 4 more seats than the previous row.

(a) Write an equation that can be used to find the number of seats in the nth row.

$$a_1 = 15 \qquad a_n = a_1 + d(n-1)$$
$$d = 4 \qquad \boxed{a_n = 15 + 4(n-1)}$$

(b) Find the number of seats in the fifth row.

$$a_5 = 15 + 4(5-1)$$
$$= 15 + 4(4)$$
$$= 15 + 16 = 31$$

$$\boxed{a_5 = 31} \leftarrow \text{seats}$$

(c) If the last row has 83 seats, how many rows are in the theater?

$$a_n = 15 + 4(n-1)$$
$$83 = 15 + 4(n-1)$$
$$83 = 15 + 4n - 4$$
$$83 = 11 + 4n$$
$$\underline{-11 \qquad -11}$$
$$\frac{72}{4} = \frac{4n}{4}$$
$$\boxed{n = 18}$$

$$a_n = 15 + 4(n-1)$$
$$a_{18} = 15 + 4(18-1)$$
$$a_{18} = 83 \checkmark$$

$$\underline{18 \text{ rows}}$$

2) The height (in feet) of the water in a tank each hour after opening its drain can be estimated by the sequence displayed in the table below.

✗	Hours (n)	1	2	3	4	5	6	7
✓	Water Height (a_n)	18	15	12	9			

(a) Write an explicit formula that represents the arithmetic sequence.

$$a_1 = \underline{18}$$
$$d = \underline{-3}$$

$$a_n = a_1 + d(n-1)$$
$$a_n = 18 - 3(n-1)$$

(b) Find the seventh term. What does this value represent in the context of the situation?

$$a_7 = 18 - 3(7-1)$$
$$a_7 = 18 - 3(6)$$
$$a_7 = 18 - 18$$
$$a_7 = 0$$

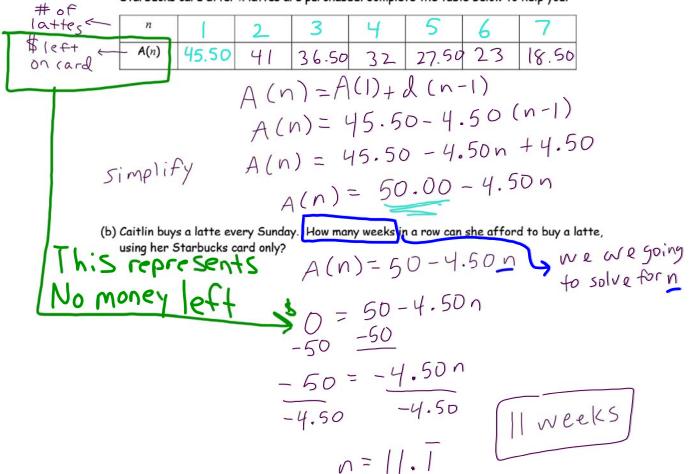The tank is empty.

(c) Would the eighth term apply in this situation? Explain.

No, we can't have a negative amount of water.

(d) Simplify the explicit formula from part (a). Compare and contrast both formulas.

subscript notation $\{a_n\}$

function notation →

Simplify in the space below ⇩	Original Formula	Simplified Formula
$a(n) = 18 - 3(n-1)$ $a(n) = 18 - 3n + 3$ $\boxed{a(n) = 21 - 3n}$	$a(n) = 18 - 3(n-1)$	$a(n) = \boxed{21} - 3n$
	comparison (same) • still have -3 (common diff) or change • still have n <u>contrast</u> • starting point is different	

(e) What was the height of the water in the tank before the drain was opened?

21 ft

3) Caitlin is given a Starbucks card worth $50. After she purchases a latte, the card's value is $45.50. After she purchases a second latte, its value is $41.

(a) Assuming the pattern continues, write an equation $A(n)$, the amount of money on the Starbucks card after n lattes are purchased. Complete the table below to help you.

# of lattes ←

$ left on card ←

n	1	2	3	4	5	6	7
$A(n)$	45.50	41	36.50	32	27.50	23	18.50

$$A(n) = A(1) + d(n-1)$$
$$A(n) = 45.50 - 4.50(n-1)$$
$$A(n) = 45.50 - 4.50n + 4.50$$

Simplify

$$A(n) = 50.00 - 4.50n$$

(b) Caitlin buys a latte every Sunday. How many weeks in a row can she afford to buy a latte, using her Starbucks card only?

This represents No money left

$$A(n) = 50 - 4.50n \longrightarrow \text{we are going to solve for } n$$

$$0 = 50 - 4.50n$$
$$-50 \quad -50$$
$$\frac{-50}{-4.50} = \frac{-4.50n}{-4.50}$$

$$n = 11.\overline{1}$$

$\boxed{11 \text{ weeks}}$

CONCLUSION!

Arithmetic sequences represent linear relationships. These sequences can be defined using the formula __$a(n) = a(1) + d(n-1)$__. In the formula, a_1 represents the first __term__ and __d__ represents the common difference. These sequences can also be defined using the linear function rule $a_n = mn + b$ where __m__ represents the common difference and __b__ represents the term <u>before the first</u>.

Homework!

1) a) Write an equation for the nth term of the arithmetic sequence. -7, -8.5, -10, -11.5, ...

    b) Using your formula, find a_{12}.

2) The first row of a dominos display has 10 dominos. Each row after the first has two more dominos than the row before it.

    a) Write the first five terms of the sequence that represents the number of dominos in each row.

    b) Write an equation that can be used to find the number of dominos in the nth row.

    c) Find the number of dominos in the 15th row.

    d) What row has 60 dominos in it?

3) During a science experiment, the temperature of a liquid substance increased 2°F every hour. After the first hour, the temperature was 56°F. Carry and Carl each wrote an equation that can be used to find the temperature of the substance after the nth hour.

Carry's Equation

$a_n = 56 + 2(n - 1)$

Carl's Equation

$a_n = 2n + 54$

(a) Are the equations equivalent? Justify your response.

(b) What does 56 represent in Carry's equation?

(c) What does 54 represent in Carl's equation?

(d) What does the coefficient 2 represent in both equations?