

Name: _____

Date: _____

INTRODUCTION TO POLYNOMIALS

Let's jump right into an exercise!

Exercise #1: Write each of the following numbers as a sum of multiples of powers of 10. The first is done as an example.

$500 + 60 + 3$

(a) $563 = 500 + 60 + 3$
 $= 5 \cdot 100 + 6 \cdot 10 + 3 \cdot 1$
 $= 5 \cdot 10^2 + 6 \cdot 10 + 3$

(b) 274 $= 200 + 70 + 4$
 $2 \cdot 100 + 7 \cdot 10 + 4$
 $2 \cdot 10^2 + 7 \cdot 10 + 4$

(c) 3,842 $= 3000 + 800 + 40 + 2$
 $3 \cdot 1000 + 8 \cdot 100 + 4 \cdot 10 + 2$
 $3 \cdot 10^3 + 8 \cdot 10^2 + 4 \cdot 10 + 2$

(d) 5,081
 $5000 + 80 + 1$
 $5 \cdot 1000 + 8 \cdot 10 + 1$
 $5 \cdot 10^3 + 8 \cdot 10 + 1$

(e) 21,478
 $20000 + 1000 + 400 + 70 + 8$
 $2 \cdot 10000 + 1 \cdot 1000 + 4 \cdot 100 + 7 \cdot 10 + 8$
 $2 \cdot 10^4 + 1 \cdot 10^3 + 4 \cdot 10^2 + 7 \cdot 10 + 8$

*We can use algebra to replace the base of 10 with a generic base of x (or whatever variable you like).

The base of a polynomial certainly doesn't have to be 10. But, all polynomials have a form similar to your answer in letter (b). Let's define them a little more definitively.

POLYNOMIAL EXPRESSIONS

Any expression of the form: $ax^n + bx^{n-1} + cx^{n-2} + \dots + \text{constant}$, where the exponents, $n, n-1, n-2$, etcetera are all positive integers. Note that **not all powers** need to be presents because the **coefficients**, i.e. a, b, c , etcetera can be zero.

$\star 2x^4 + 1x^3 + 4x^2 + 7x + 8$
the exponent can't be negative

Exercise #3: Of the expressions shown below, circle all of them that represent polynomials. Discuss why the ones that aren't polynomials fail the definition above.

✓ $4x^2 + 8x + 1$
Yes!

$9x^2 + 2x + \frac{1}{x}$
 $9x^2 + 2x + x^{-1}$
Not a polynomial.

↓ ↓ ↓
 $2^x + 3^x + 4^x$
Not a polynomial.
We can't have a variable in the exponent.

✓ $2x^2 + 5x^3 - x + 8$
Yes!



It is often important to place polynomials in their STANDARD FORM. The standard form of a polynomial is simply achieved by writing it as an **equivalent expression** where the powers on the variables ALWAYS DESCEND.

Exercise #4: Write each of the following polynomials in standard form.

(a) $3x^2 + 5x^3 + 7 - 8x$ \rightarrow (b) $9x^4 + 2x - x^2 + 1$ (c) $3 - 2x - 5x^2$
 $5x^3 + 3x^2 - 8x + 7$ $9x^4 - x^2 + 2x + 1$ $-5x^2 - 2x + 3$

Polynomials are simply abstract representations of numbers that we see every day and they behave like these numbers as well. Let's look at adding polynomials together.

Try this!

(a) Add

$5x^2 + 2x + 3$
$+ 2x^2 + 7x + 1$
<hr/>
$7x^2 + 9x + 4$

$7x^2 + 9x + 4$

(b) Find the sum of the polynomials $-4x^2 + 9x - 3$ and $7x^2 - 5x + 4$.

$$\begin{array}{r} -4x^2 + 9x - 3 \\ + 7x^2 - 5x + 4 \\ \hline 3x^2 + 4x + 1 \end{array}$$

Finding sums of polynomials is fairly easy. Subtracting them, though, can lead to a lot of errors.

Exercise #5: Find each of the following differences. Be careful and rewrite as an equivalent addition problem if necessary.

(a) $6x^2 + 5x + 3$
 $- 2x^2 - 4x + 7$

(b) $(4x^2 - 2x + 7) - (-2x^2 + x - 3)$

$$\begin{array}{r} 4x^2 - 2x + 7 + 2x^2 - x + 3 \\ \hline 6x^2 - 3x + 10 \end{array}$$

Exercise #6: For each of the following, write an equivalent polynomial in simplest standard form.

(a) $6x^2 + 2x - 3 - x^2 + 4x - 1$

$$5x^2 + 6x - 4$$

(b) $6x^2 + 2x - 3 - (x^2 + 4x - 1)$

$$\begin{array}{r} 6x^2 + 2x - 3 - x^2 - 4x + 1 \\ \hline 5x^2 - 2x - 2 \end{array}$$

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**INTRODUCTION TO POLYNOMIALS
HOMEWORK**

1. Write each of the following integers as multiples of powers of 10. The first is done as a reminder of this process.

(a) 563

(b) 278

(c) 703

$$563 = 500 + 60 + 3$$

$$= 5 \cdot 100 + 6 \cdot 10 + 3$$

$$= 5 \cdot 10^2 + 5 \cdot 10 + 3$$

(d) 5,378

(f) 19,073

2. Which of the following would be the value of the expression $5x^3 + 2x^2 + 8x + 4$ when $x = 10$?

(1) 6,432

(3) 5,284

(2) 2,854

(4) 528

3. Which of the following would be the value of the expression $8x^3 + 2x + 3$ when $x = 10$?

(1) 823

(3) 8,203

(2) 8,023

(4) 8,230

4. Which of the following is *not* a polynomial expression?

(1) x^4 (3) $1 - 2x^3$ (2) 3^x (4) $6x + 1$ 

There's more on the next page!

5. Write each of the following polynomial expressions in standard form.

(a) $7x^2 + 4x^3 + 5 + 2x$

(b) $4 - x - 5x^2$

(c) $x^3 + x - 7x^2 + 2$

(d) $2x + 1 - 3x^3 + 5x^2$

(e) $4x^3 - 2x^2 + 6 - 8x$


(f) $y^5 + y^{10} - y^2 + y^7$

6. Find each of the following sums and differences. Write your answer in simplest standard form.


(a) $6x^2 - 2x + 8 + 3x^2 + 7x - 2$

(b) $x^3 + 4x^2 - 8x + 3 + x^3 - x + 1$

(c) $(5x^2 + 3x - 1) - (3x^2 - 6x + 4)$



(d) $(2x^3 - 5x^2 + 8x - 1) - (-4x^3 + 8x^2 - 3x - 9)$



(e) $4x^2 + 6x - 3 - 3x^2 + 2x + 4$

(f) $(4x^2 + 6x - 3) - (3x^2 + 2x + 4)$

