Name: ______________________________ Date: ________________

COMPLETING THE SQUARE

The turning point of a parabola and its general shape are relatively easy to determine if the quadratic function is written in its **shifted or vertex form**. Review this in the first exercise.

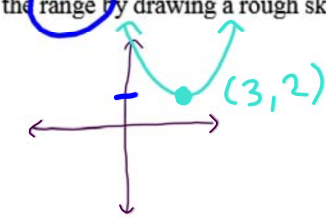
Do Now: Given the function $y=(x-3)^2+2$ do the following.

(a) Give the coordinates of the turning point.

$(3,2)$

(b) Determine the range by drawing a rough sketch.

$[2,\infty)$



Vertex Form: opposite x-coordinate, y- coordinate

The question then is how we take a quadratic of the form $y=ax^2+bx+c$ and put it into its shifted form. This procedure is known as **Completing the Square**. But, it needs some additional review.

Exercise #2: Write each of the following as an equivalent trinomial.

(a) $(x+5)^2$

$\frac{1}{2}b=5$

$(x+5)(x+5)$

$x^2+5x+5x+25$

$\boxed{x^2+10x+25}$

$a=1$
$b=10$
$c=25$

(b) $(x-1)^2$

$\frac{1}{2}b=-1$

$(x-1)(x-1)$

$x^2-1x-1x+1$

$\boxed{x^2-2x+1}$

$a=1$
$b=-2$
$c=1$

(c) $(x+4)^2$

$\frac{1}{2}b=4$

$(x+4)(x+4)$

$x^2+4x+4x+16$

$\boxed{x^2+8x+16}$

$a=1$
$b=8$
$c=16$

Exercise #3: Given each trinomial in Exercise #2 of the form x^2+bx+c, what is true about the relationship between the value of b and the value of c? Illustrate.

$\frac{1}{2}(\overset{b}{10})=(5)^2$

$=\underset{c}{25}$

Half of the b term, squared, is the c term.

Exercise #4: Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.

(a) $x^2+20x+100$

$(x+10)^2$

(b) x^2-6x+9

$(x-3)^2$

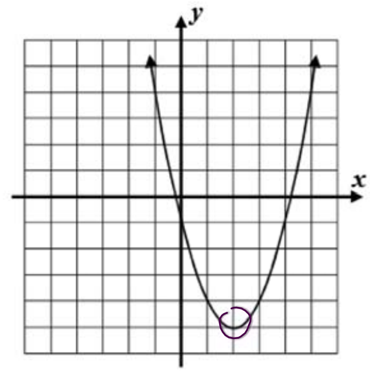
(c) x^2+2x+1

$(x+1)^2$

We are finally ready to learn the method of **Completing the Square**. This method has many uses, but the one we will work with today is to manipulate equations of quadratics from their **standard form** to their **vertex form**.

☆

***Exercise* #5:** The quadratic $\boxed{y=x^2-4x-1}$ is shown graphed below.



(a) Consider only the binomial x^2-4x. What would you need to add on to it to create a perfect square trinomial? (See Exercise #3).

$x^2-4x\boxed{+4}-1$

(b) In order to add zero to the binomial x^2-4x, what should we subtract to offset adding 4 to make it a perfect square?

$x^2-4x+4-4-1$

(c) Use the Method of Competing the Square now to rewrite the trinomial x^2-4x-1 in an equivalent, shifted form. According to this form, what are the coordinates of the vertex? Verify by examining the graph.

$x^2-4x+4-4-1$

VERTEX FORM ☆ $y=(x-2)^2-5$ turning point → $(2,-5)$

This procedure is what is known as an **algorithm**. In other words, we follow a recipe. Here it is: IT → "b" term

COMPLETING THE SQUARE

① HALF IT ③ SHARE IT
② SQUARE IT

For the quadratic $y=x^2+bx+c$ (note that $a=1$).

1. Find half of the value of b, i.e. $\frac{b}{2}$
2. Square it, i.e. $\left(\frac{b}{2}\right)^2$
3. Add and subtract it

There is nothing like practice on these.

***Exercise* #6:** Write each quadratic in **vertex form** by Completing the Square. Then, identify the quadratic's turning point. The last two problems will involve fractions. Stick with it!

(a) $y=x^2+6x-2$

$y=x^2+6x-2$

$y=x^2+6x+9-9-2$

$y=(x+3)^2-11$

T.P. → $(-3,-11)$

(b) $y=x^2-10x+27$

$y=x^2-10x+25-25+27$

$y=(x-5)^2+2$

turning point $(5,2)$