

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**ROOTS AND IRRATIONAL NUMBERS: SQUARE ROOTS**

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Square roots, cube roots, and higher level roots are important mathematical tools because they are the **inverse operations** to the operations of **squaring and cubing**. In this unit we will study these operations, as well as numbers that come from using them. First, some basic review of what you've seen before.

**Do Now:** Find the value of each of the following **square roots**. Write a reason for your answer in terms of a multiplication equation.

(a)  $\sqrt{25} = \pm 5$

(b)  $\sqrt{9}$

(c)  $\sqrt{100}$

$5 \cdot 5 = 25$   
 $-5 \cdot -5 = 25$

$\pm 3$

$\pm 10$

(d)  $\sqrt{0}$

0

(e)  $\sqrt{\frac{1}{4}} = \pm \frac{1}{2}$

$\sqrt{\frac{1}{4}}$

(f)  $\sqrt{\frac{64}{9}} = \pm \frac{8}{3}$

It is generally agreed upon that all **positive, real numbers** have two square roots, a positive one and a negative one. We simply designate which one we want by either including a negative sign or leaving it off.

**Exercise #2:** Given the function  $f(x) = \sqrt{x+3}$ , which of the following is the value of  $f(46)$ ?

(1) 22

(3) 16

(2) 5

(4) 7

$f(46) = \sqrt{46+3}$   
 $= \sqrt{49} = \pm 7$

Square roots have an interesting property when it comes to multiplication. We will discover that property in the next exercise.

**Exercise #3:** Find the value of each of the following products.

(a)  $\sqrt{4} \cdot \sqrt{9} =$

$2 \cdot 3 = 6$

(b)  $\sqrt{4 \cdot 9} = \sqrt{36} = 6$

(c)  $\sqrt{4} \cdot \sqrt{25} =$

$2 \cdot 5 = 10$

(d)  $\sqrt{4 \cdot 25} = \sqrt{100}$

$= 10$

What you should notice in the last exercise is the following important property of square roots.

<b>MULTIPLICATION PROPERTY OF SQUARE ROOTS</b>		
★ 1. $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$	likewise	★ 2. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

One obvious use for this is to multiply two “unfriendly” square roots to get a nice result.

**Exercise #5:** Find the result of each of the following products.

(a) $\sqrt{2} \cdot \sqrt{8} =$ $\sqrt{16} = \pm 4$	(b) $\sqrt{12} \cdot \sqrt{3} =$ $\sqrt{36} = \pm 6$	(c) $\sqrt{20} \cdot \sqrt{5} =$ $\sqrt{100} = 10$
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One less obvious use for the square root property above is in **simplifying square roots of non-perfect squares**. This is a fairly antiquated skill that is almost completely irrelevant to algebra, but it often arises on standardized tests and thus is a good skill to become fluent with.

**Exercise #6:** To introduce **simplifying square roots**, let’s do the following first.

(a) List out the first 10 perfect squares (starting with 1).

$1, 4, 9, 16, 25, 36, 49, 64, 81, 100$

(b) Now consider  $\sqrt{18}$ . Which of these perfect squares is a factor (divides) of 18?

(c) Simplify the  $\sqrt{18}$ . This is known as writing it in **simplest radical form**.

$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$$

The key to simplifying any square root is to find the **largest perfect square** that is a factor of the **radicand** (the number under the square root).

**Exercise #7:** Write each of the following square roots in simplest radical form.

(a) $\sqrt{8}$ $\sqrt{4} \cdot \sqrt{2}$ $2\sqrt{2}$	(b) $\sqrt{45}$ $\sqrt{9} \cdot \sqrt{5}$ $3\sqrt{5}$	(c) $\sqrt{48}$ $\sqrt{16} \cdot \sqrt{3}$ $4\sqrt{3}$
(d) $-\sqrt{75}$ $-\sqrt{25} \sqrt{3}$ $-5\sqrt{3}$	(e) $\sqrt{72}$	(f) $-\sqrt{500}$

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**SQUARE ROOTS  
HOMEWORK**

1. Simplify each of the following. Each will result in a rational number answer. You can check your work using your calculator, but should try to do all of them without it.

(a)  $\sqrt{36}$

(b)  $-\sqrt{4}$

(c)  $\sqrt{121}$

(d)  $\sqrt{\frac{1}{9}}$

(e)  $-\sqrt{100}$

(f)  $\sqrt{\frac{81}{36}}$

(g)  $-\sqrt{\frac{1}{16}}$

(h)  $-\sqrt{144}$

2. Find the final, simplified answer to each of the following by evaluating the square roots first. Show the steps that lead to your final answers.

(a)  $\sqrt{9} + \sqrt{25} - \sqrt{64}$

(b)  $5\sqrt{4} + 2\sqrt{81}$

$3 + 5 - 8$   
 $8 - 8 = 0$

(c)  $\frac{2\sqrt{25} + 2}{3}$

(d)  $\sqrt{\frac{1}{4}}(\sqrt{121} - \sqrt{9})$

All of the square roots so far have been "nice." We will discuss what this means more in the next lesson. We can use the Multiplication Property to help simplify certain products of not-so-nice square roots.

3. Find each of the following products by first multiplying the **radicands** (the numbers under the square roots).

(a)  $\sqrt{2} \cdot \sqrt{50} =$

(b)  $\sqrt{3} \cdot \sqrt{12} =$

(c)  $5\sqrt{6} \cdot \sqrt{24} =$

(d)  $\sqrt{25} - \sqrt{2} \cdot \sqrt{8} =$

(e)  $\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{18}} =$

(f)  $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{27}{4}} =$

4. Write each of the following in **simplest radical form**. Show the work that leads to your answer. The first exercise has been done to remind you of the procedure.

$$\begin{aligned} \text{(a) } \sqrt{162} &= \\ &= \sqrt{81} \cdot \sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$

$$\text{(b) } \sqrt{20} =$$

$$\text{(c) } -\sqrt{90} =$$

$$\text{(d) } \sqrt{48} =$$

$$\text{(e) } -\sqrt{8} =$$

$$\text{(f) } \sqrt{300} =$$

5. Write each of the following products in **simplest radical form**. The first is done as an example for you.

$$\begin{aligned} \text{(a) } 3\sqrt{12} &= \\ &= 3 \cdot \sqrt{4} \cdot \sqrt{3} \\ &= 3 \cdot 2 \cdot \sqrt{3} \\ &= 6\sqrt{3} \end{aligned}$$

$$\text{(b) } 4\sqrt{45} =$$

$$\text{(c) } \frac{1}{2}\sqrt{32} =$$

$$\text{(d) } -2\sqrt{288} =$$

$$\text{(e) } \frac{\sqrt{108}}{3} =$$

$$\text{(f) } \frac{-\sqrt{320}}{16} =$$