Name: _____ Date: _____

1
4
9
16
25
36
49
64
81
100
. . .

FINDING ZEROES BY COMPLETING THE SQUARE

In the last lesson, we saw how to find the **zeroes** of a **quadratic function** if it was in **vertex or shifted form**. Do a warm-up problem to refresh this equation solving technique.

***Exercise* #1:** For the quadratic function $y = 2(x-2)^2 - 36$.

(a) Find the zeroes in simplest radical form.

(b) Find the zeroes to the nearest *tenth*.

① $2-3\sqrt{2} = -2.2$

② $2+3\sqrt{2} = 6.2$

REMEMBER: How do we find zeros?

$2(x-2)^2 - 36 = 0$

$+36 \quad +36$

$\frac{2(x-2)^2}{2} = \frac{36}{2}$

$\sqrt{(x-2)^2} = \sqrt{18}$

$x-2 = \sqrt{18}$

$x-2 = \sqrt{9}\sqrt{2}$

$x-2 = \pm 3\sqrt{2}$

$+2 \quad +2$

$x = 2 \pm 3\sqrt{2}$

But, of course, in order for us to find the zeroes using inverse operations as in (a), we need our quadratic in the form $y = a(x-h)^2 + k$. In order to do this, we will use our technique of **Completing the Square**.

***Exercise* #2:** Consider the quadratic $y = x^2 - 6x - 16$.

(a) Find the zeroes of this function by factoring.

(b) Find the zeroes of this function by Completing the Square.

In order to do this, follow these steps:
1. Move the constant to the right of the equal sign.
2. Complete the square.

A M

$-8 \cdot 2 = -16$

$-8+2 = -6$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x-8=0 \quad x+2=0$

$+8 \quad -2 \quad -2$

$x = 8 \quad x = -2$

$3+5 = 8$

$3-5 = -2$

$\frac{-6}{2} = (-3)^2 = 9$

$x^2 - 6x - 16$

$x^2 - 6x + 9 - 9 - 16 = 0$

$(x-3)^2 - 25 = 0$

$+25 \quad +25$

$\sqrt{(x-3)^2} = \sqrt{25}$

$x-3 = \pm 5$

$+3 \quad +3$

$x = 3 \pm 5$

Let's practice!

1. Which equation has the same solutions as $x^2 + 6x - 7 = 0$?
   1) $(x+3)^2 = 2$
   2) $(x-3)^2 = 2$
   3) $(x-3)^2 = 16$
   4) $(x+3)^2 = 16$

$x^2+6x-7=0$

$\frac{6}{2}=(3)^2$

$=9$

$x^2+6x+9-9-7=0$

$(x+3)^2-2$

$(x-3)^2-2$

$(x-3)^2-16$

$(x+3)^2-16$

$(x+3)^2-16=0$

$+16 \quad +16$

$(x+3)^2=16$

2. When solving the equation $x^2 - 8x - 7 = 0$ by completing the square, which equation is a step in the process?
   1) $(x-4)^2 = 9$
   2) $(x-4)^2 = 23$
   3) $(x-8)^2 = 9$
   4) $(x-8)^2 = 23$

$\frac{-8}{2}=(-4)^2$

$=16$

$x^2-8x+16-16-7=0$

$(x-4)^2-23=0$

$+23 \quad +23$

$(x-4)^2=23$

3. Solve the equation $x^2 - 6x = 15$ by completing the square.

4. If $4x^2 - 100 = 0$, the roots of the equation are
   1) –25 and 25
   2) –25, only
   3) –5 and 5
   4) –5, only

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FINDING ZEROES BY COMPLETING THE SQUARE
HOMEWORK

1. Solve the equation $x^2 - 4x - 12 = 0$ two ways:

   (a) By Factoring (b) By Completing the Square

   ( )( )

2. Solve the equation $x^2 + 10x + 21 = 0$ two ways:

   (a) By Factoring (b) By Completing the Square

3. Find the solutions to the following equation in simplest radical form by using the technique of Completing the Square.

$$x^2 + 8x - 2 = 0$$