

Name: _____

Date: _____

UNIT 2 LESSON 1: EQUATIONS AND THEIR SOLUTIONS

A LOT of time is spent in Algebra learning how to solve equations and then solving them for various purposes. So, it goes without saying that we really need to understand what it means for something to “solve” an equation. First, let’s make sure we understand what an equation is:

EQUATION DEFINITION

An equation is simply a statement about the **equality** of two expressions. In other words, anything that takes this form:

Expression #1 = Expression #2

Do Now: Which of the following is **not** an equation?

(1) $3 + 1 = 4 + 0$

(3) $2(4x + 1)$

(2) $x^2 - 2x = 8$

(4) $1 + 3 = 5$

Equations have equal signs

Equations can be either true, like (1) above, or false, like (4) above, depending on whether the two expressions are equal (true) or not equal (false).

Exercise #1: Consider the equation $2x - 8 = 10 - x$.

(a) Why can't you determine whether this equation is true or false?

We don't know what x is.

(b) If $x = 5$, will the equation be true? How can you tell?

$2x - 8 = 10 - x$
 $2(5) - 8 = 10 - (5)$
 $10 - 8 = 10 - 5$
 $2 = 5$ X
 Not equal !!

(c) Show that $x = 6$ makes the equation true. Remember to think very carefully always about your order of operations.

$2x - 8 = 10 - x$
 $2(6) - 8 = 10 - (6)$
 $12 - 8 = 10 - 6$
 $4 = 4$ ✓

The solution to this is 6 because they are equal

SOLUTIONS TO EQUATIONS

A value for a variable is called a **solution to the equation** if, when substituted into both expressions, results in the equation being true.

Exercise #2: Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $2x+3=17$ and $x=7$

$$\begin{aligned} 2(7)+3 &= 17 \\ 14+3 &= 17 \\ 17 &= 17 \checkmark \\ \text{TRUE} \end{aligned}$$

(b) $\frac{x-20}{5} = -4$ and $x=10$

$$\begin{aligned} \frac{(10)-20}{5} &= -4 \\ \frac{-10}{5} &= -4 \quad -2 \neq -4 \quad \times \\ \text{FALSE} \end{aligned}$$

(c) $2(x+5)=6(x-1)$ and $x=4$

$$\begin{aligned} 2(4+5) &= 6(4-1) \\ 2(9) &= 6(3) \\ 18 &= 18 \checkmark \\ \text{TRUE} \end{aligned}$$

(d) $x^2-1=2x+2$ and $x=-1$

$$\begin{aligned} (-1)^2-1 &= 2(-1)+2 \\ 1-1 &= -2+2 \\ 0 &= 0 \checkmark \\ \text{TRUE} \end{aligned}$$

(e) $\frac{3(x+2)}{4}-1=5$ and $x=2$

$$\begin{aligned} \frac{3(2+2)}{4}-1 &= 5 \rightarrow 3-1=5 \\ \frac{3(4)}{4}-1 &= 5 \quad 2=5 \\ \frac{12}{4}-1 &\neq 5 \quad \times \text{ FALSE} \end{aligned}$$

(f) $\frac{3}{4}x-1=-\frac{1}{2}x+9$ and $x=8$

$$\begin{aligned} \frac{3}{4}(8)-1 &= -\frac{1}{2}(8)+9 \\ 6-1 &= -4+9 \\ 5 &= 5 \checkmark \text{ TRUE} \end{aligned}$$

You try: Kirk was checking to see if $x=7$ was a solution to the equation $4x-3=2x+11$. He concluded that it was not a solution based on the following work. Was he correct?

$$\begin{aligned} 4x-3 &= 2x+11 \\ 4 \cdot 7 - 3 &= 2 \cdot 7 + 11 \\ 4 \cdot 4 &= 2 \cdot 18 \\ 16 &= 36 \quad \text{No!} \end{aligned}$$

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**EQUATIONS AND THEIR SOLUTIONS
HOMEWORK**

1. Decide if each of the following are **equations** or **expressions**. You do not need to solve the equations or evaluate the expressions.

(a) $5x + 13$

(b) $4x + 3 = 12$

(c) $\frac{6(x-1)}{4} + 1 = 5$

(d) $3(x+2)^2 - (45)^3$

(e) $3^2 - 5|2x - 15|$

(f) $3[(x+2)^2 + 2(x-4)] = 3\sqrt{4(2x+1)}$

2. Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $x - 4 = 12$ and $x = 8$

(b) $\frac{(3+x)}{4} = 3$ and $x = 9$

(c) $(x+2) - 3(x-4) = 6$ and $x = 4$

(d) $\frac{1}{3}(x+2) = -\frac{2}{5}(x-9)$ and $x = 4$