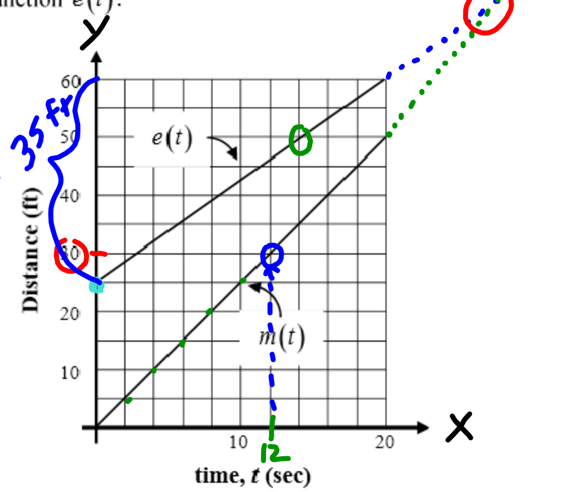
Name: ______________________ Date: ____________

Average Rate of Change – Yes!

Functions are rules that give us **outputs** when we supply them with **inputs**. Very often, we want to know how **fast** the outputs are changing compared to a change in the input values. This is referred to as the **average rate of change** of a function.

Do Now: Max and his younger sister Evie are having a race in the backyard. Max gives his sister a head start and they run for 20 seconds. The distance they are along in the race, in feet, is given below with Max's distance given by the function $m(t)$ and Evie's distance given by the function $e(t)$.



(a) How do you interpret the fact that $m(12) = 30$? Illustrate your response by using the graph.

At 12 seconds, Max has traveled 30 ft.

(b) If both runners start at $t = 0$, how much of a head start does Max give his little sister? How can you tell?

25 ft head start

(c) Will Max catch up to his sister? How can you tell?

Yes, because $m(t)$ is steeper than $e(t)$.

(d) How far does Max run during the 20 second race? How far does Evie run? What calculation can you do to find Evie's distance?

50ft

60 ft − 25 ft

$\boxed{35 \text{ ft}}$

(e) How fast do both Evie and Max travel? In other words, how many feet do each of them run per second? Express your answers as decimals and attach units.

Max's Speed (feet per second)

$\frac{5 \text{ ft}}{2 \text{ sec}} = 2.5$ ft/sec

Evie's Speed (feet per second)

$\frac{25 \text{ ft}}{14 \text{ sec}} = 1.785$ ft/sec

In the first exercise we were calculating the **rate** that the **function's output (y-values)** were changing compared to the **function's input (or x-values)**. This is known as finding the **average rate of change** of the function. You might think you've seen this before. And you have.

***Exercise* #1:** Finding the average rate of change is the same as finding the SLOPE of a line.

There is, of course, a formula for finding average rate of change. Let's get it out of the way.

b and a → inputs

AVERAGE RATE OF CHANGE

For the function $y = f(x)$, the average rate that $f(x)$ changes from $x = a$ to $x = b$ is given by:

$$\frac{\Delta y}{\Delta x} \qquad \frac{f(b) - f(a)}{b - a}$$

Consider the function given by $\underbrace{\boxed{f(x) = x^2 + 3}}$ RULE. Find its average rate of change from $\boxed{x = -1 \text{ to } x = 3}$ INPUTS (a, b). Carefully show the work that leads to your final answer.

$$\frac{12-4}{3-(-1)} = \frac{8}{4} = \boxed{2}$$

$$\frac{f(b)-f(a)}{b-a}$$

$f(b) = x^2+3$

$f(3) = (3)^2+3$
$9+3$
12

$f(a) = x^2+3$

$f(-1) = (-1)^2+3$
$1+3=4$

$f(b)=12$ $b=3$
$f(a)=4$ $a=-1$

$$\frac{f(a)-f(b)}{a-b}$$

$$\frac{4-12}{-1-3} = \frac{-8}{-4} = \boxed{2}$$

***Exercise* #2:** The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$? Show the calculations that lead to your answer.

(1) $-\frac{3}{2}$

(2) $\frac{6}{4}$

(3) $-\frac{7}{6}$

(4) -1

x	$h(x)$
0	10
2	9
4	6
6	3

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HW

AVERAGE RATE OF CHANGE HOMEWORK

1. Frances is selling glasses of lemonade. The function $g(t)=\frac{t^2+4}{2}$ models the number of glasses she has sold, g, after t-hours. What is the average rate at which she is selling lemonade between $t=2$ and $t=6$ hours. Include proper units in your answer.

2. Consider the function given by $f(x)=9-x^2$. Find its average rate of change between the following points. Carefully show the work that leads to your final answer.

(a) $x=0$ to $x=3$ (b) $x=-1$ to $x=5$ (c) $x=-2$ to $x=2$

3. The function $f(x)$ is given in the table below. Find its average rate of change between the following points. Show the calculations that lead to your answer.

(a) $x=-3$ to $x=1$

(b) $x=0$ to $x=4$.

x	$f(x)$
-3	7
0	-2
1	3
4	-8