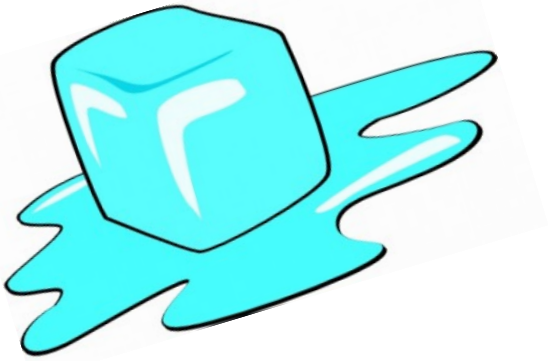
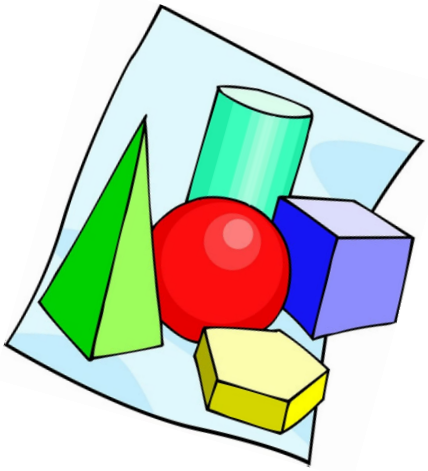
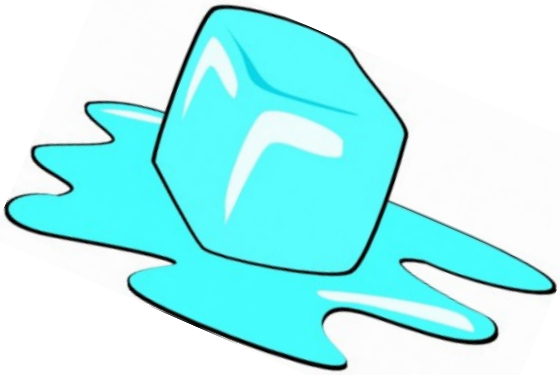




# GEOMETRY (COMMON CORE)

## FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM



# Geometry [Common Core] Regents Exam Study Guide

## Polygons – Interior/Exterior Angles

**Sum of Interior Angles: 180 (n - 2)**

**Each Interior Angle of a Regular Polygon:** $\frac{180\ (n-2)}{n}$

**Sum of Exterior Angles: 360°**

**Each Exterior Angle:** $\frac{360}{n}$

## Triangles

**Classifying Triangles**

**Sides:**

Scalene: No congruent sides

Isosceles: 2 congruent sides

Equilateral: 3 congruent sides

**Angles:**

Acute: All angles are < 90°

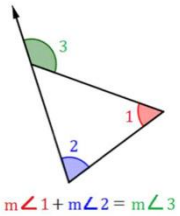
Right: One right angle that is 90°

Obtuse: One angle that is > 90°

Equiangular: 3 congruent angles (60°)

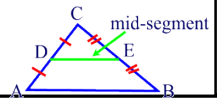
**All triangles have 180°**

**Exterior Angle Theorem:** The exterior angle is equal to the sum of the two non-adjacent interior angles.



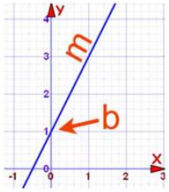

**Midsegment:** segment joining the midpoints

- Always parallel to the third side
- $\frac{1}{2}$ the length of the third side
- Splits the triangle into two similar triangles.



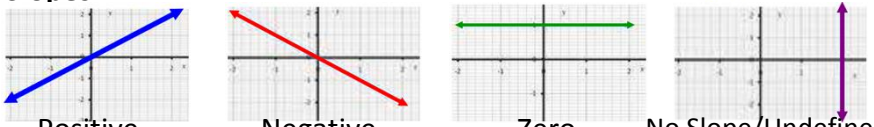

## Coordinate Geometry

**Standard Form of a Line:** $y = mx + b$ ,where m is the slope and b is the y-intercept.




**Slope Formula:** $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

**Slopes:**



Positive | Negative | Zero | No Slope/Undefined

**Parallel** Lines have the **SAME** slope

**Perpendicular** Lines have **NEGATIVE RECIPROCAL** slopes (flip & change the sign)

**Collinear Points** are points that lie of the **same** line.

**Midpoint Formula:** $M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

**Distance Formula:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Segment Ratios:**

$\frac{x - x_1}{x_2 - x} = Given\ Ratio$ $\frac{y - y_1}{y_2 - y} = Given\ Ratio$

## Pythagorean Theorem

To find the missing side of any **right** triangle, use:

$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs, and *c* is the hypotenuse

## Isosceles Triangle

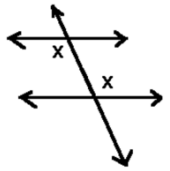
- 2 ≅ sides and 2 ≅ base angles
- The altitude drawn from the vertex is also the median and angle bisector
- If two sides of a triangle are ≅, then the angles opposite those ≅ sides are ≅.

## Triangle Inequality Theorems

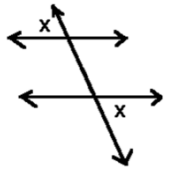
- The sum of 2 sides must be greater than the third side
- The difference of 2 sides must be less than the third side
- The longest side is opposite the largest angle
- The shortest side is opposite the smallest angle

## Parallel Lines

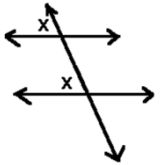
**Alternate Interior** angles are **congruent**



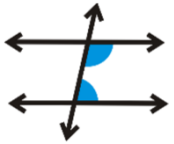

**Alternate Exterior** angles are **congruent**




**Corresponding** angles are **congruent**

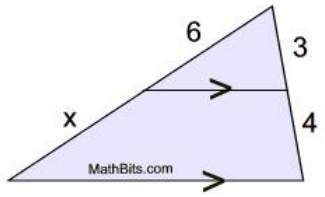



**Same-Side Interior** angles are **supplementary**



## Side – Splitter Theorem

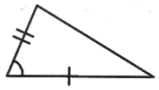
If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.



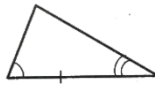
## Triangle Congruence Theorems

Side-Side-Side **(SSS)**

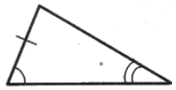
Side-Angle-Side **(SAS)**



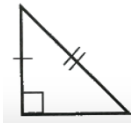
Angle-Side-Angle **(ASA)**



Angle-Angle-Side **(AAS)**



Hypotenuse-Leg **(HL)**



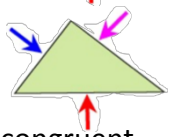
**CPCTC –** Corresponding Parts of Congruent Triangles are Congruent

## Similar Triangle Theorems

Angle-Angle (**aa**)

Side-Angle-Side **(SAS)**

Side-Side-Side **(SSS)**

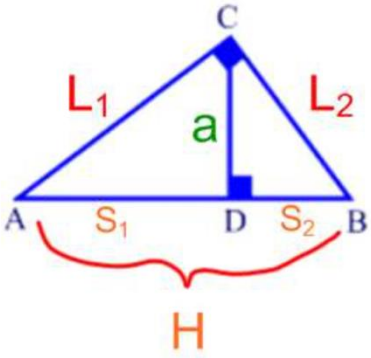


- Similar figures have congruent angles and proportional sides
- **CSSTP**-Corresponding Sides of Similar Triangles are in Proportion
- In a proportion, the product of the means equals the product of the extremes

## Geometric Mean Theorems

**Altitude Theorem (SAAS / Heartbeat Method):**
The altitude is the geometric mean between the 2 segments of the hypotenuse.

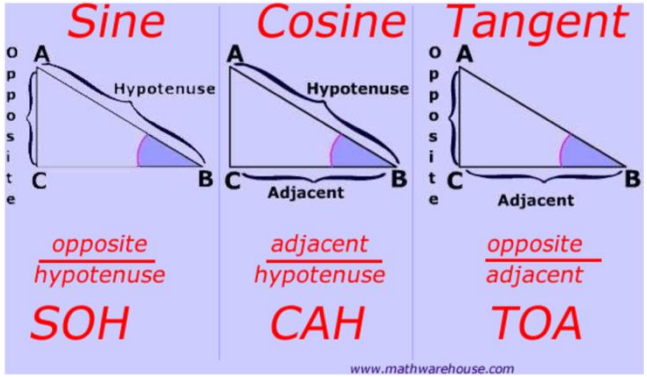
$$\frac{S_1}{a} = \frac{a}{S_2}$$




**Leg Theorem (HYLLS / PSSW):**
The leg is the geometric mean between the segment It touches and the whole hypotenuse.

$$\frac{S_1}{L_1} = \frac{L_1}{H} \quad \text{and} \quad \frac{S_2}{L_1} = \frac{L_2}{H}$$

## Trigonometry




- When solving for a side, use the *sin*, *cos*, and *tan* buttons
- When solving for an angle, use the $sin^{-1}$, $cos^{-1}$, and $tan^{-1}$ buttons

**Cofunctions:**

- Sine and Cosine are cofunctions, which are complementary

$$\boldsymbol{\sin\theta = \cos(90° - \theta)}$$
$$\boldsymbol{\cos\theta = \sin(90° - \theta)}$$

- If $\angle A$ and $\angle B$ are the acute angles of a right triangle, then $\boldsymbol{\sin A = \cos B}$

## Factoring

**The order of Factoring:**

**Greatest Common Factor (GCF)**

↓

**Difference of Two Perfect Squares (DOTS)**

↓

**Trinomial (TRI)**

**GCF:**

$$\boldsymbol{ab + ac = a(b + c)}$$

**DOTS:**

$$\boldsymbol{x^2 - y^2 = (x + y)(x - y)}$$

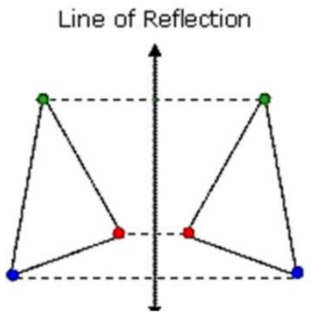
**TRI:**

$$\boldsymbol{x^2 - x + 6 \text{ » } (x + 2)(x - 3)}$$

## Transformational Geometry

**Rigid Motion**: transformations that preserve distance, congruency, angle measure, and shape.

### Reflection – FLIP




$r_{x-axis}(x, y) = (x, -y)$

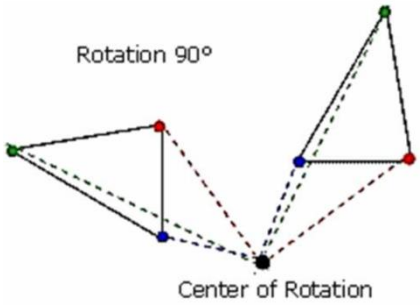
$r_{y-axis}(x, y) = (-x, y)$

$r_{y=x}(x, y) = (y, x)$

$r_{y=-x}(x, y) = (-y, -x)$

$r_{(o,o)}(x, y) = (-x, -y)$

### Rotation – TURN

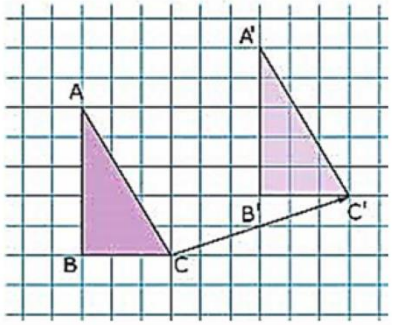



$R_{90°}(x, y) = (-y, x)$

$R_{180°}(x, y) = (-x, -y)$

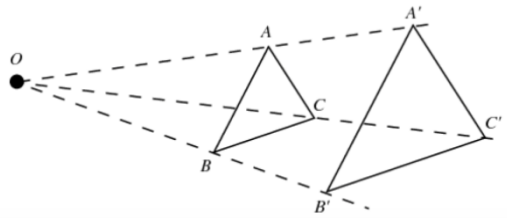
$R_{270°}(x, y) = (y, -x)$

### Translation – SHIFT/MOVE




$T_{a,b}(x, y) = (x + a, y + b)$

### Dilation – ENLARGEMENT/REDUCTION




$D_k(x, y) = (k \cdot x, k \cdot y)$

- **Dilations create similar figures**
- **Dilations are NOT rigid motions, since they do NOT preserve distance.**

### Composition of Transformations

**When you see "∘", work from right to left.**

$$R_{90°} \circ T_{3,-4}$$

Do this Second!

Do this First!

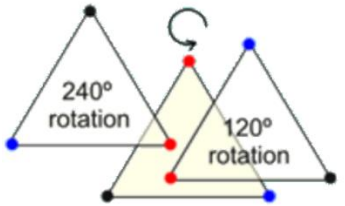
Translation, followed by a Rotation.

### Types of Composition Transformations

- **A composition of 2 reflections over 2 parallel lines is equivalent to a TRANSLATION.**
- **A composition of 2 reflections over 2 intersecting lines is equivalent to a ROTATION.**

### Rotational Symmetry Theorem

**A regular polygon with $n$ sides always has rotational symmetry, with rotations in increments equal to its central angle of $\frac{360°}{n}$**




equilateral triangle

# Circles

## Equations

**General/Standard Equation of a Circle:**

$$x^2 + y^2 + Cx + Dy + E = 0$$

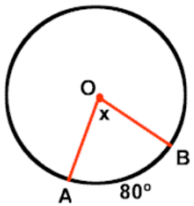
where *C, D*, and *E* are constants

**Center – Radius Equation of a Circle:**

$$(x-h)^2 + (y-k)^2 = r^2$$

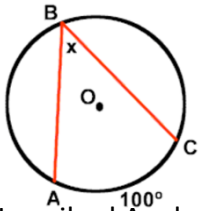
where *(h, k)* is the center and *r* is the radius.

## Angles



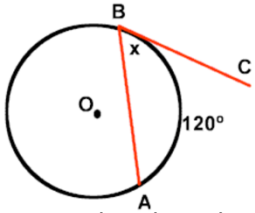

Central Angle:

$\measuredangle x = \widehat{AB}$



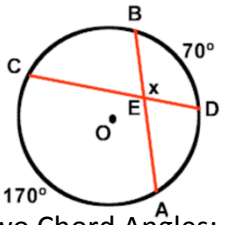

Inscribed Angle:

$\measuredangle x = \frac{1}{2}\widehat{AC}$



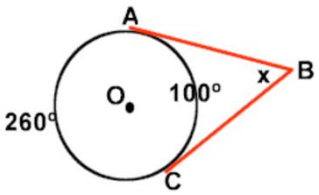

Tangent-Chord Angle:

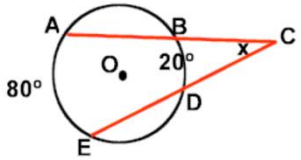
$\measuredangle x = \frac{1}{2}\widehat{AC}$

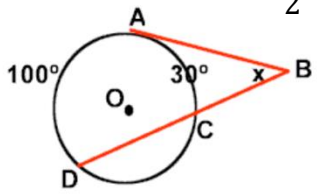



Two Chord Angles:

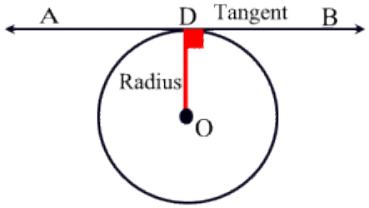
$\measuredangle x = \frac{\widehat{Arc_1} + \widehat{Arc_2}}{2}$



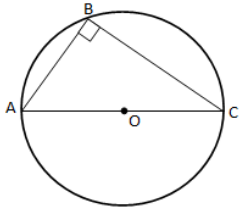



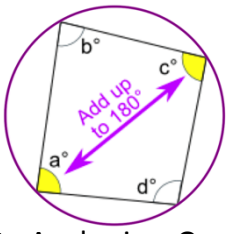

$$\frac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$$




Tangent-Radius Angle = 90°

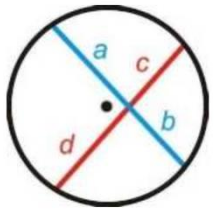



Angle Inscribed in a semicircle = 90°

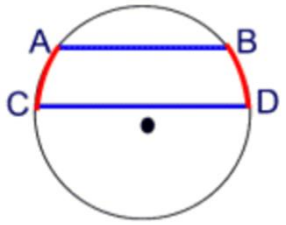



Opposite Angles in a Quad. = 180°

## Segments

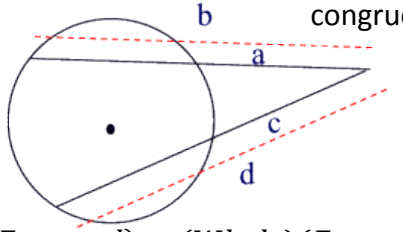



$(a)(b) = (c)(d)$



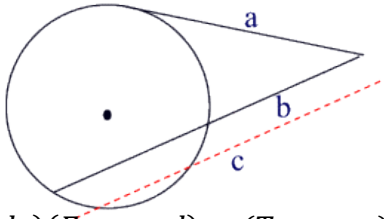

$\overline{AB} \parallel \overline{CD}$, $\widehat{AC} \cong \widehat{BD}$

Parallel chords intercept congruent arcs



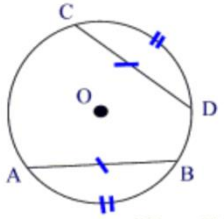

$(Whole)(External) = (Whole)(External)$

*(W)(E) = (W)(E)*

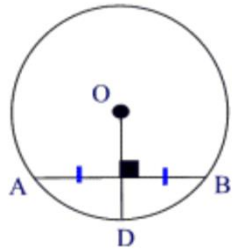



$(Whole)(External) = (Tangent)^2$

*(W)(E) = (T)²*

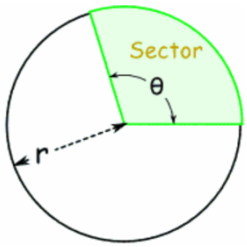



$\overline{CD} \cong \overline{AB}$; $\widehat{CD} \cong \widehat{AB}$



## Circles (Con't)

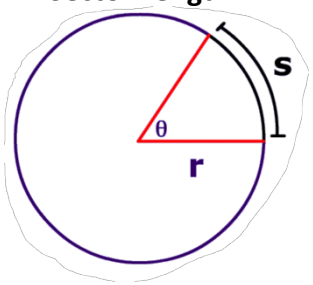
### Area of a Sector




$$A = \frac{1}{2}r^2\theta$$

where $A$ is the area of the sector, $r$ is the radius, and $\theta$ is an angle in radians.

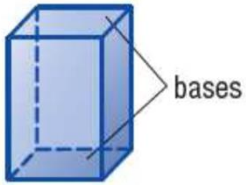
### Sector Length



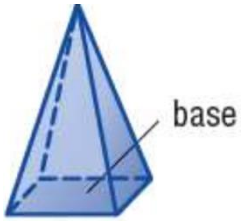

$$s = r\theta$$

where $s$ is the sector length, $r$ is the radius, and $\theta$ is an angle in radians.

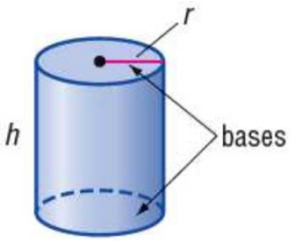
## 3D Figures



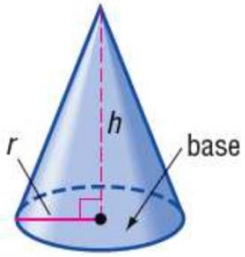

Prism



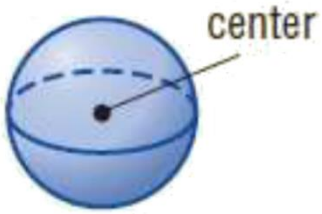

Pyramid




Cylinder

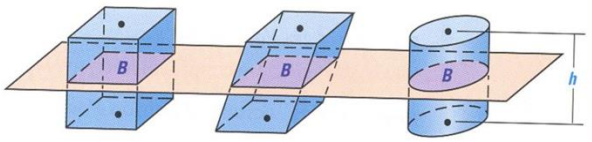



Cone

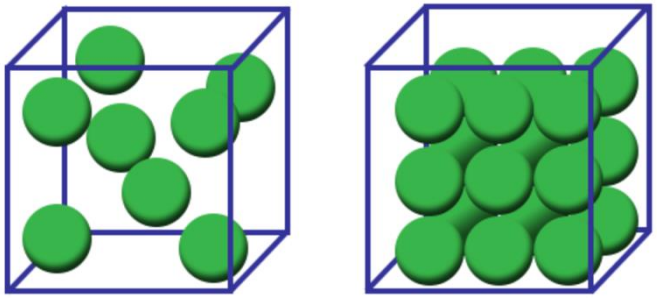



Sphere

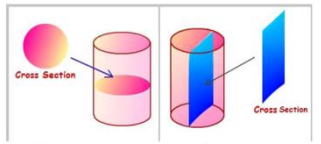
**Cavalieri's Principle:** If two solids have the same height and the same cross-sectional area at every level, then the solids have the same volume.

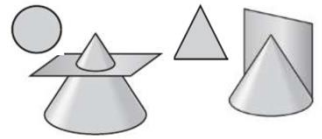



$$\boldsymbol{Mass} = (\boldsymbol{Density}) \cdot (\boldsymbol{Volume})$$

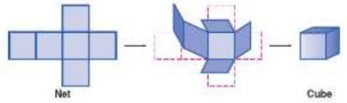


### Cross Sections

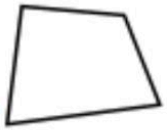


### Nets

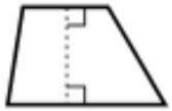


## The Quadrilateral Family Tree



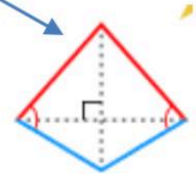
Quadrilateral

- four-sided polygon
- sum of interior angles = 360°



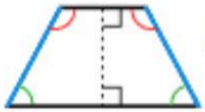
Trapezoid (US)

- at least 1 pair of || sides



Kite

- exactly two distinct pairs of adjacent ≅ sides
- diagonals are ⊥
- one diagonal divides the quadrilateral into 2 ≅ triangles



Isosceles Trapezoid

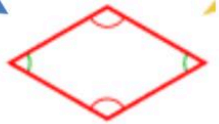
- each pair of base angles are ≅
- diagonals are ≅
- one pair of ≅ sides, the legs (not the parallel sides which are called the bases)

Parallelogram

- opposite sides are ||
- opposite sides are ≅
- opposite angles are ≅
- consecutive angles = 180°
- diagonals bisect each other

Rectangle

- all angles are right angles
- diagonals are ≅



Rhombus

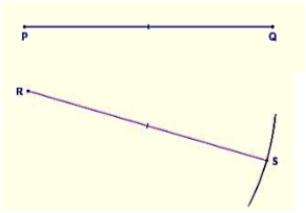
- all sides are ≅
- diagonals are ⊥ (perpendicular)
- diagonals bisect opposite angles

Square

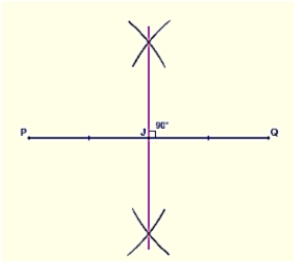
- all angles are right angles
- diagonals are ≅
- all sides are ≅
- diagonals are ⊥
- diagonals bisect opposite angles

## Basic Constructions

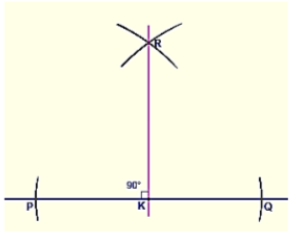
### Copy a line segment



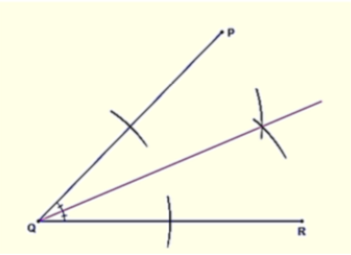

### Perpendicular Bisector



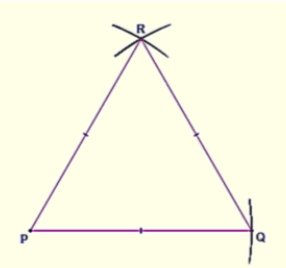

### Perpendicular Line thru a point on a line



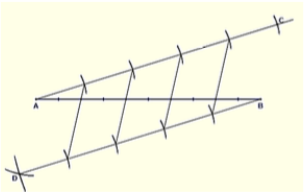

### Bisect an angle



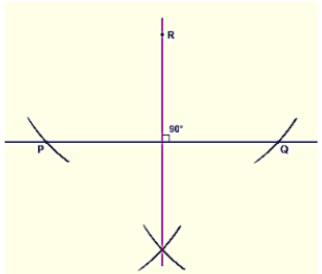

### Equilateral Triangle



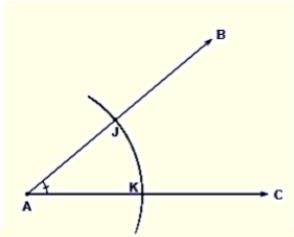

### Dividing a segment into equal parts

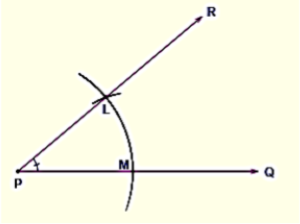


### Perpendicular line thru an external point

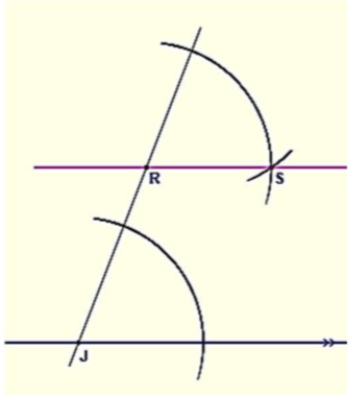



### Copy an angle

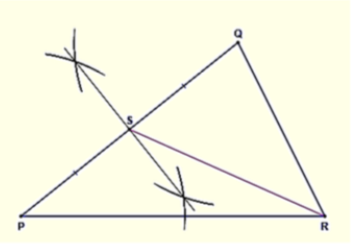


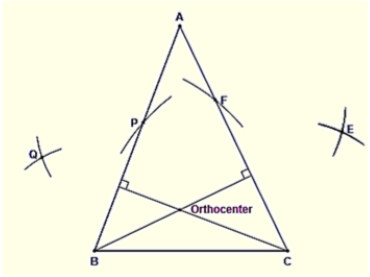

### Parallel line



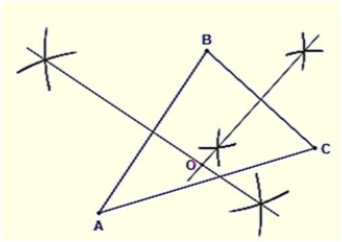

### Median – vertex to midpoint




### Orthocenter - altitudes

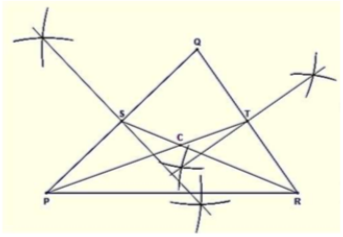



### Circumcenter – perpendicular bisectors




- Equidistant to each vertex of the triangle
- Used to circumscribe a circle

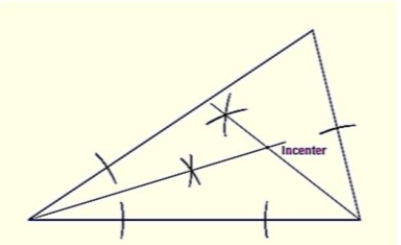
### Centroid - medians




- 2:1 ratio

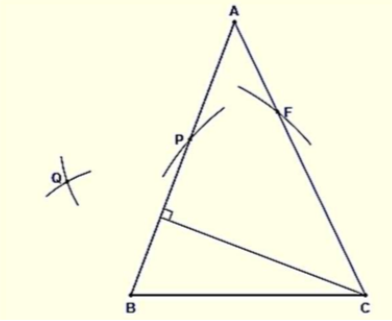
$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

### Incenter – angle bisectors




- equidistant to each side of the triangle
- Used to inscribe a circle

### Altitude – vertex perpendicular to opposite side




### Inscribed square in a circle